Math 1552
Summer 2023
Quiz 1
May 25
Time limit: 20 Minutes

Name (Print): Key

Canvas email: ______

Teaching Assistant/Section: ______

GT ID: | | | | | | | | | |

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Sign Your Name: Sal

Please clearly organize your work, show all steps, simplify all answers, and BOX your answers.

1. (5 points) Give the **general** anti-derivative of the following function:

$$f(x) = \frac{1}{2\sqrt{x}} - e^{x/5} + \frac{1}{\sqrt{9-x^2}}$$

$$f(x) = \frac{1}{2}x^{-1/2} - e^{x/5} - \frac{1}{\sqrt{9(1-(\frac{x}{3})^2)}} = \frac{1}{2}x^{-1/2} - e^{x/5} - \frac{1}{3}\frac{1}{\sqrt{1-(\frac{x}{3})^2}}$$

$$F(x) = \frac{1}{2}\cdot\frac{x^{-1/2+1}}{-1/2+1} - 5e^{x/5} - \frac{1}{3}\cdot 3\sin^{-1}(x/3) + C$$

$$= \boxed{\sqrt{x} - 5e^{x/5} - \sin^{-1}(x/3) + C}$$

2. (5 points) Suppose $f(x)$ is an even function and $g(x)$ is an odd function. If $\int_{-2}^{2} f(x)\,dx = 5$ and $\int_{-2}^{0} g(x)\,dx = 2$, find $\int_0^2 f(x) - g(x)\,dx$.

$$\int_0^2 f(x) - g(x)\,dx = \int_0^2 f(x)\,dx - \int_0^2 g(x)\,dx$$

① $\int_{-2}^{2} f(x)\,dx = 2\cdot\int_0^2 f(x)\,dx$

② $\int_{-2}^{0} g(x)\,dx = -\int_0^2 g(x)\,dx$

$$= \frac{1}{2}(5) - (-2)$$

$$= \frac{5}{2} + 2 = \frac{5}{2} + \frac{4}{2} = \boxed{9/2}$$

3. (10 points) Suppose $f(x) = (x+1)^2$. Use a general Riemann Sum

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

to evaluate the definite integral of $f(x)$ on the interval [-1,3], by following these steps:

(a) Find the length of each subinterval Δx in terms of n. $\Delta x = \frac{b-a}{n} = \frac{3-(-1)}{n}$ $\boxed{4/n}$

(b) Evaluate x_k^* as the right-hand endpoint of the subinterval. $\boxed{-1 + \frac{4k}{n}}$

$x_k = a + k\Delta x = -1 + 4k/n$

(c) Evaluate the function at x_k^*, i.e. find $f(x_k^*)$. *Simplify.* $\boxed{16k^2/n^2}$

$$f(x_k) = \left(-1 + \frac{4k}{n} + 1\right)^2 = \frac{16k^2}{n^2}$$

(d) Using the following summation formulas to simplify the sigma notation, find an expression for $R_n = \sum_{k=1}^{n} f(x_k^*)\Delta x$ that does not involve sigma's.

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ $\qquad$ $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

$$R_n = \sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} \frac{16k^2}{n^2}\cdot\frac{4}{n} = \frac{64}{n^2}\sum_{k=1}^{n} k^2 = \frac{64}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}$$

$$= \boxed{\frac{32}{3}\,\frac{n(n+1)(2n+1)}{n^3}}$$

(e) Using the sum you found in the previous step, find the definite integral.

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{32}{3}\,\frac{n(n+1)(2n+1)}{n^3} = \frac{32}{3}\cdot\frac{2}{1} = \boxed{\frac{64}{3}}$$

Check w/ FTC & u-sub

$$\int_{-1}^{3} (x+1)^2\,dx = \int_0^4 u^2\,du = \frac{1}{3}u^3\Big|_0^4 = \frac{1}{3}\cdot 64 - \frac{1}{3}\cdot 0 = 64/3 \checkmark$$

$u = x+1$

$du = dx$