**Math 1552**
**Summer 2023**
**Quiz 1** Practice
**May 25, 2023**
**Time limit: 20 Minutes**

**Name (Print):** Key

**Canvas email:**

Teaching Assistant/Section:

**GT ID:**

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Sign Your Name: ______________

Please clearly organize your work, show all steps, simplify all answers, and BOX your answers.

1. (5 points) Give the **general** anti-derivative of the following function:

$$f(x) = 2\sec x(\tan x - \sec x)$$

$f(x) = 2\sec x\tan x - 2\sec^2 x$

$F(x) = \boxed{2\sec x - 2\tan x + C}$

2. (5 points) Suppose $f(x)$ is an even function and $g(x)$ is an odd function. If $\int_0^3 f(x)\,dx = 5$ and $\int_0^3 g(x)\,dx = 2$, find $\int_{-3}^3 f(x) + g(x)\,dx$.

$f$ is even so $\int_{-3}^{3} f(x)\,dx = 2\cdot\int_0^3 f(x)\,dx = 2\cdot 5 = 10$

$g$ is odd so $\int_{-3}^{3} g(x)\,dx = 0$

Therefore,

$$\int_{-3}^{3} f(x) + g(x)\,dx = \int_{-3}^{3} f(x)\,dx + \int_{-3}^{3} g(x)\,dx = 10 + 0 = \boxed{10}$$

Check ans w/ FTC.

3. (10 points) Suppose $f(x) = x^2 + 1$. Use a general Riemann Sum

$$\int_{-1}^{2} x^2+1\,dx = \frac{1}{3}x^3 + x \Big|_{-1}^{2}$$

$$= (8/3 + 2) - (\tfrac{-1}{3} - 1)$$

$$= 3 + 2 + 1$$

$$= 6$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

to evaluate the definite integral of $f(x)$ on the interval [-1,2], by following these steps:

**(a)** Find the length of each subinterval $\Delta x$ in terms of $n$. $\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{n}$ $\boxed{3/n}$

**(b)** Evaluate $x_k^*$ as the right-hand endpoint of the subinterval. $\boxed{-1 + 3k/n}$

$$x_k = a + k\Delta x = -1 + k \cdot 3/n$$

**(c)** Evaluate the function at $x_k^*$, i.e. find $f(x_k^*)$. *Simplify.* $\boxed{\frac{9k^2}{n^2} - \frac{6k}{n} + 2}$

$$f(x_k^*) = f(-1 + 3k/n) = \left(-1 + \frac{3k}{n}\right)^2 + 1$$

$$= \frac{9k^2}{n^2} - \frac{6k}{n} + 1 + 1 = \frac{9k^2}{n^2} - \frac{6k}{n} + 2$$

**(d)** Using the following summation formulae, find $\sum_{k=1}^{n} f(x_k^*)\Delta x$

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ $\qquad$ $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

$$R_n = \sum_{k=1}^{n} f(x_k^*)\Delta x = \sum_{k=1}^{n} \left(\frac{9k^2}{n^2} - \frac{6k}{n} + 2\right)\left(\frac{3}{n}\right)$$

$$= \sum_{k=1}^{n} \frac{27k^2}{n^3} - \frac{18k}{n^2} + \frac{6}{n} = \frac{27}{n^3}\sum_{k=1}^{n} k^2 - \frac{18}{n^2}\sum_{k=1}^{n} k + \frac{6}{n}\sum_{k=1}^{n} 1$$

$$= \boxed{\frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{18}{n^2}\frac{n(n+1)}{2} + \frac{6}{\cancel{n}} \cdot \cancel{n}}$$

**(e)** Using the sum you found in the previous step, find the definite integral.

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{27}{6}\frac{n(n+1)(2n+1)}{n^3} - \frac{18}{2}\frac{n(n+1)}{n^2} + 6$$

$$= \frac{27}{6} \cdot 2 - 9 + 6 = 9 - 9 + 6 = \boxed{6}$$