Math 1552
Summer 2023
Quiz 2 *QUP only*
May 25
Due date: Sunday at 11:59PM

Name (Print): _____

Canvas email: _____

Teaching Assistant/Section: _____

Key

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: _____ Jal

For Question (0.) below please list any **outside resources** you used to help solve quiz problems. You can use calculators, texbook/course documents, websites, solving tools, or each other (e.g., TI-89 calculator, textbook formula sheet on page 281, 3Blue1Brown YouTube video on integrals, WolframAlpha, Symbolab). **Be specific.** List the name of anyone who helped you. If you used no outside resources, write `N/A`.

As always, anything you submit must be your own work. Never submit the work of someone else.

Please clearly organize your work, show all steps, simplify all answers, and BOX your answers.

0. (1 point) *Full credit for accurately following the directions above.*

N/A

1. (5 points) Compute $F'(x)$ using the fundamental theorem of calculus.

$$F(x) = \int_x^{\sqrt{x}} \sqrt{t^2+1}\, dt$$

$G(x) = \int_0^x \sqrt{t^2+1}\, dt$

$\Rightarrow G'(x) = \sqrt{x^2+1}$ by FTC

$F(x) = G(\sqrt{x}) - G(x)$

$\Rightarrow F'(x) = G'(\sqrt{x}) \cdot (\sqrt{x})' - G'(x)$

$= \sqrt{(\sqrt{x})^2+1} \cdot \frac{1}{2\sqrt{x}} - \sqrt{x^2+1} = \boxed{\frac{\sqrt{x}+1}{2\sqrt{x}} - \sqrt{x^2+1}}$

2. (4 points) Use u-substitution to find the general anti-derivative of $f(x) = \frac{\cos\left(\frac{1}{x}\right)}{x^2}$.

$\int \frac{1}{x^2}\cos\left(\frac{1}{x}\right) dx$ (u over $\frac{1}{x}$, $-du$ over $\frac{1}{x^2}dx$)

u-sub Box:
$u = \frac{1}{x}$
$du = \frac{-1}{x^2} dx$
$-du = \frac{1}{x^2} dx$

$= \int -\cos(u)\, du = -\sin(u) + C$

$= \boxed{-\sin\left(\frac{1}{x}\right) + C}$

3. (10 points) In this problem you will find the area bounded the curves $y = f(x) = 2x^2 - 2x$ and $y = g(x) = x^3 - x^2$ by following these steps:

(a) Find the x-values of the intersections points of the curves. *Separate values with commas.*

Set $y=y$

$2x^2 - 2x = x^3 - x^2 \Rightarrow x(x-2)(x-1) = 0$

$\Rightarrow x^3 - 3x^2 + 2x = 0 \Rightarrow x = 0, 1, 2$

$\Rightarrow x(x^2 - 3x + 2) = 0$

$x =$ 0, 1, 2

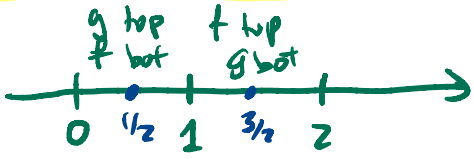
(b) Determine the bounded intervals where $f(x)$ or $g(x)$ is on top/bottom.

f on top (1, 2) g on top (0, 1)

$f(\frac{1}{2}) = 2\cdot\frac{1}{4} - 2\cdot\frac{1}{2} = -\frac{1}{2}$

$f(\frac{3}{2}) = 2\cdot\frac{9}{4} - 3 = 1.5$

$g(\frac{1}{2}) = \frac{1}{8} - \frac{1}{4} = \frac{-1}{4}$ $g(\frac{3}{2}) = (\frac{3}{2})^3 - (\frac{3}{2})^2 = \frac{27}{8} - \frac{9}{4} = \frac{9}{8} = 1.125$



(c) Set up integrals to find the area for each region between the curves. *Do not evaluate.*

Area 1: $\int_0^1 (x^3 - x^2) - (2x^2 - 2x)\, dx$

Area 2: $\int_1^2 (2x^2 - 2x) - (x^3 - x^2)\, dx$

(d) Finally, find the area by evaluating the integrals you set up from part (c) and adding the areas together.

Area 1: $\int_0^1 x^3 - 3x^2 + 2x\, dx = \frac{1}{4}x^4 - x^3 + x^2 \Big|_0^1 = (\frac{1}{4} - 1 + 1) - 0 = \frac{1}{4}$ //

Area 2: $\int_1^2 -x^3 + 3x^2 - 2x\, dx = -\frac{1}{4}x^4 + x^3 - x^2 \Big|_1^2 = (-\frac{1}{4}\cdot 16 + 8 - 4) - (-\frac{1}{4} + 1 - 1)$

$= -4 + 8 - 4 + \frac{1}{4} = \frac{1}{4}$ //

total $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$