Math 1552
Summer 2023
Quiz 2 Practice
May 25
Time limit: 20 Minutes

Name (Print): Key

Canvas email:

Teaching Assistant/Section:

GT ID:

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Sign Your Name: Jal

Please clearly organize your work, show all steps, simplify all answers, and $\boxed{\text{BOX}}$ your answers.

1. (5 points) Compute $F'(x)$ using the fundamental theorem of calculus.

$$F(x) = \int_{x^2}^{2x} \frac{\sqrt{t}}{t^2 - 1}\, dt$$

$G(x) = \int_0^x \frac{\sqrt{t}}{t^2-1}\, dt$

$G'(x) = \frac{\sqrt{x}}{x^2-1}$ (by FTC)

$F(x) = G(2x) - G(x^2)$

$\Rightarrow F'(x) = G'(2x) * (2x)' - G'(x^2)(x^2)'$

$= \frac{\sqrt{2x}}{(2x)^2-1} * 2 - \frac{\sqrt{x^2}}{(x^2)^2-1}(x^2)' = \boxed{\frac{\sqrt{x}}{\sqrt{2}(x^2-1)} - \frac{2|x|x}{x^4-1}}$

2. (4 points) Use u-substitution to find the general anti-derivative of $f(x)$.

$$f(x) = \frac{1}{\sqrt{x}e^{-\sqrt{x}}} \sec(e^{\sqrt{x}} + 1)\tan(e^{\sqrt{x}} + 1)$$

$\int \underbrace{\frac{1}{\sqrt{x}} e^{\sqrt{x}}}_{2\,du} \sec(\underbrace{e^{\sqrt{x}} + 1}_{u}) \tan(\underbrace{e^{\sqrt{x}}+1}_{u})\, \underbrace{dx}$

u-sub BOX

$u = e^{\sqrt{x}} + 1$

$du = e^{\sqrt{x}} (\sqrt{x})'\, dx$

$du = \frac{1}{2\sqrt{x}} e^{\sqrt{x}}\, dx$

$2\,du = \frac{1}{\sqrt{x}} e^{\sqrt{x}}\, dx$

$= \int \sec u \tan u * 2\, du$

$= 2\sec u + C = \boxed{2\sec(e^{\sqrt{x}} + 1) + C}$

3. (10 points) In this problem you will find the area between the curves $y = f(x) = x^3 + x^2$ and $y = g(x) = 2x^2 + 6x$ by following these steps:

(a) Find the x-values of the intersections points of the curves. *Separate values with commas.*

Set $y = y$

$x^3 + x^2 = 2x^2 + 6x$

$\Rightarrow x^3 - x^2 - 6x = 0$

$\Rightarrow x(x^2 - x - 6) = 0$

$\Rightarrow x(x-3)(x+2) = 0$

$x = 3, -2, 0$

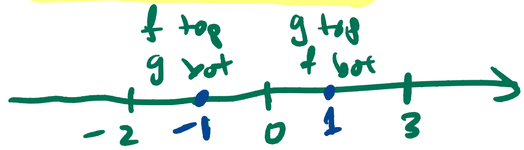
$x = \boxed{-2, 0, 3}$

(b) Determine the intervals where $f(x)$ or $g(x)$ is on top/bottom. *Separate intervals with* $\cup$.

f on top $\boxed{(-2, 0)}$ g on top $\boxed{(0, 3)}$

$f(-1) = -1 + 1 = 0$ $g(-1) = 2 - 6 = -4$

$f(1) = 2$ $g(1) = 8$



(c) Set up integrals to find the area for each region between the curves. *Do not evaluate.*

Area 1: $\displaystyle\int_{-2}^{0} (x^3 + x^2) - (2x^2 + 6x)\, dx$

Area 2: $\displaystyle\int_{0}^{3} (2x^2 + 6x) - (x^3 + x^2)\, dx$

(d) Finally, find the area by evaluating the integrals you set up from part (c) and adding the areas together.

Area 1: $\displaystyle\int_{-2}^{0} x^3 - x^2 - 6x\, dx = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2 \Big|_{-2}^{0}$

$= 0 - \left(\frac{1}{4}\cdot 16 - \frac{1}{3}(-8) - (12)\right)$

$= -4 + \frac{8}{3} + 12 = 8 - \frac{8}{3} = 16/3$

Area 2: $\displaystyle\int_{0}^{3} -x^3 + x^2 + 6x\, dx = -\frac{1}{4}x^4 + \frac{1}{3}x^3 + 3x^2 \Big|_{0}^{3}$

$= -\frac{1}{4}\cdot 81 + 9 + 27$

$= 36 - \frac{81}{4} = \frac{144 - 81}{4} = \frac{63}{4}$

Total

$\frac{16}{3} + \frac{63}{4} = \frac{64 + 189}{12} = \boxed{\frac{253}{12}}$