Math 1552
Summer 2023
Quiz 3
May 25
Time limit: 20 Minutes

Name (Print): Key

Canvas email:

Teaching Assistant/Section:

GT ID: | | | | | | | | | |

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: Sal

Please clearly organize your work, show all steps, simplify all answers, and $\boxed{\text{BOX}}$ your answers.

1. (4 points) Fill in the blanks using arbitrary constants $A, B, C, D, \ldots$ (as many as you need) to set up a partial fraction decomposition for the given rational function. Leave any unused boxes blank. *Do not integrate!*

$$\frac{x^3-8}{x^3(x^2+1)^2} = \boxed{\frac{A}{x}} + \boxed{\frac{B}{x^2}} + \boxed{\frac{C}{x^3}} + \boxed{\frac{Dx+E}{x^2+1}} + \boxed{\frac{Fx+G}{(x^2+1)^2}}$$

2. (8 points) Use partial fractions to find the general anti-derivative of $f(x) = \dfrac{2x+1}{x^2-7x+6}$.

$$\frac{2x+1}{x^2-7x+6} = \frac{2x+1}{(x-6)(x-1)} = \frac{A}{x-6} + \frac{B}{x-1} \Rightarrow 2x+1 = A(x-1)+B(x-6)$$

$$= Ax - A + Bx - 6B$$

$$= (A+B)x + (-A-6B)$$

So $A+B=2$ & $-A-6B=1$ $\Rightarrow$ $-5B=3$ so $B=-3/5$, $A=13/5$

$$\int \frac{2x+1}{x^2-7x+6}\,dx = \int \frac{13/5}{x-6} + \frac{-3/5}{x-1}\,dx = \boxed{\frac{13}{5}\ln|x-6| - \frac{3}{5}\ln|x-1| + C}$$

3. (8 points) Evaluate.

$$\int \frac{1}{x^2\sqrt{x^2-1}}\,dx = \int \frac{1}{\sec^{\cancel{2}}\theta\,\underbrace{\sqrt{\sec^2\theta-1}}_{\tan^2\theta}} \cdot \cancel{\sec\theta}\tan\theta\,d\theta$$

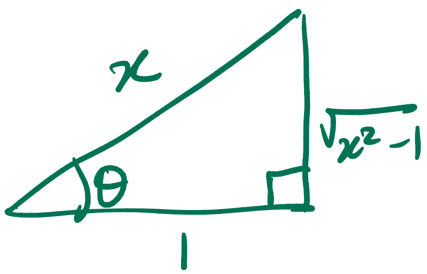
$\sec^2\theta - 1 = \tan^2\theta$

trig sub box

$$\boxed{\begin{aligned} x &= \sec\theta \\ dx &= \sec\theta\tan\theta\,d\theta \end{aligned}}$$

$$= \int \frac{1}{\sec\theta\,\cancel{\tan\theta}}\,\cancel{\tan\theta}\,d\theta$$

$$= \int \frac{1}{\sec\theta}\,d\theta = \int \cos\theta\,d\theta$$



$$= \sin\theta + C$$

$$= \boxed{\frac{\sqrt{x^2-1}}{x} + C}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{x}{1}$$