Math 1552
Summer 2023
Quiz 3 Practice
May 25

Name (Print): Key

Canvas email:

Teaching Assistant/Section:

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: Sal

Please clearly organize your work, show all steps, simplify all answers, and $\boxed{\text{BOX}}$ your answers.

1. (4 points) Fill in the blanks using arbitrary constants $A, B, C, D, \ldots$ (as many as you need) to set up a partial fraction decomposition for the given rational function. Leave any unused boxes blank. *Do not integrate!*

$$\frac{x+4}{x^2(x^2+3)^2} = \boxed{\frac{A}{x}} + \boxed{\frac{B}{x^2}} + \boxed{\frac{Cx+D}{x^2+1}} + \boxed{\frac{Ex+F}{(x^2+1)^2}} + \boxed{}$$

2. (8 points) Use partial fractions to find the general anti-derivative of $f(x) = \dfrac{1}{x(x^2+1)}$.

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$\Rightarrow A(x^2+1) + (Bx+C)x = 1$$

$$\Rightarrow Ax^2 + A + Bx^2 + Cx = 1$$

$$\Rightarrow (A+B)x^2 + Cx + A = 1$$

So $A+B=0$, $C=0$, $A=1$ $\Big\}$ $A=1$, $B=-1$, $C=0$

$$\int \frac{1}{x(x^2+1)}\,dx = \int \frac{1}{x} + \frac{-x}{x^2+1}\,dx$$

$$= \ln|x| - \int \frac{x}{x^2+1}\,dx$$

$$= \ln|x| - \int \frac{1}{u}\cdot\frac{1}{2}\,du$$

$$= \boxed{\ln|x| - \ln|x^2+1| + C}$$

u-sub Box

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

3. (8 points) Evaluate.

$$\int \frac{1}{x^2\sqrt{x^2+1}}\,dx = \int \frac{1}{\tan^2\theta\sqrt{\tan^2\theta+1}}\cdot\sec^2\theta\,d\theta$$

$\tan^2\theta+1=\sec^2\theta$

trig sub Box

$$\boxed{\begin{aligned} x &= \tan\theta \\ dx &= \sec^2\theta\,d\theta \end{aligned}}$$

$$= \int \frac{1}{\tan^2\theta\underbrace{\cancel{\sqrt{\sec^2\theta}}}_{\cancel{\sec\theta}}}\sec^{\cancel{2}}\theta\,d\theta$$

$$= \int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \int \frac{1/\cos\theta}{\sin^2\theta/\cos^2\theta}\,d\theta$$

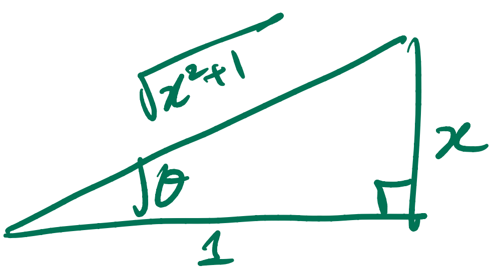
$$= \int \frac{\cos^{\cancel{2}}\theta}{\sin^2\theta}\cdot\frac{1}{\cancel{\cos}\theta}\,d\theta$$

u-sub

$$\boxed{\begin{aligned} u &= \sin\theta \\ du &= \cos\theta\,d\theta \end{aligned}}$$

$$= \int \frac{\cos\theta}{\sin^2\theta}\,d\theta = \int \frac{1}{u^2}\,du$$

$$= -\frac{1}{u}+C = \frac{-1}{\sin\theta}+C = \underline{\underline{-\csc\theta+C}}$$



$$= \boxed{-\frac{\sqrt{x^2+1}}{x}+C}$$

$\tan\theta = \frac{x}{1} = \frac{\text{opp}}{\text{adj}}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{x^2+1}}{x}$