Math 1552 Summer 2023 **Name (Print):** Key

Quiz 4 Practice **June 15**

Teaching Assistant/Section:

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Sign Your Name: Jal

Please clearly organize your work, show all steps, simplify all answers, and $\boxed{\text{BOX}}$ your answers.

1. (4 points) Find a general formula a_n for the n-th term of the sequence. You do not need to show work on this problem but please put your final answer in the box.

   Hint: be sure to include your starting value for n.

$$\frac{1}{2}, \frac{-5}{6}, \frac{9}{24}, \frac{-13}{120}, \frac{17}{720}, \ldots$$

numerators are every other odd number, and alternating

denominators are factorials

$2! = 2,\ 3! = 6,\ 4! = 24,\ 5! = 120,\ 6! = 720$

$$\boxed{\frac{(-1)^n(4n+1)}{(n+2)!},\ n \geq 0}$$

2. (10 points) Evaluate the improper integral.

$$\int_2^\infty \frac{2}{t^2-1}\,dt$$

$$\int_2^N \frac{2}{t^2-1}\,dt = \int_2^N \frac{-1}{t+1} + \frac{1}{t-1}\,dt = -\ln(t+1) + \ln(t-1)\Big|_2^N$$

$$= -\ln(N+1) + \ln(N-1) - \left(-\ln(3) + \cancel{\ln(1)}^{\,0}\right)$$

$$\frac{2}{t^2-1} = \frac{A}{t+1} + \frac{B}{t-1}$$

$A(t-1) + B(t+1) = 2$

$\to t(A+B) + (-A+B) = 2$

$A+B=0$
$-A+B=2$ $\quad B=1,\ A=-1$

$$= \ln\left(\frac{N-1}{N+1}\right) + \ln(3).$$

$$\text{So } \int_2^\infty \frac{2}{t^2-1}\,dt = \lim_{N\to\infty} \int_2^N \frac{2}{t^2-1}\,dt$$

$$= \lim_{N\to\infty} \ln\left(\frac{N-1}{N+1}\right) + \ln 3 = \boxed{\ln 3}$$

(*) $\frac{N-1}{N+1} \to 1$ as $N\to\infty$

So $\ln\left(\frac{N-1}{N+1}\right) \to \ln(1) = 0$ as $N \to \infty$

3. (6 points) For each sequence, determine the limit of the sequence as n tends to infinity. If the limit diverges, write either DNE, ∞ DNE, or $-\infty$ DNE in the box, as appropriate. You do not have to show your work for problems on this page, but please put your final answer in the box.

(a) $\left\{\left(1+\frac{2}{n}\right)^{-n}\right\}$

$\boxed{e^{-2}}$

$a_n = \left(1+\frac{2}{n}\right)^{-n}$

$b_n = \ln(a_n) = -n \cdot \ln\left(1+\frac{2}{n}\right) = \dfrac{\ln\left(1+\frac{2}{n}\right)}{-1/n}$

So $a_n \to e^{-2}$ as $n \to \infty$ since $a_n = e^{b_n} = e^{\ln(a_n)}$

$\lim\limits_{n\to\infty} b_n \underset{\text{L'Hop } \frac{0}{0}}{=} \lim\limits_{n\to\infty} \dfrac{\frac{1}{1+2/n}\left(1+\frac{2}{n}\right)'}{+1/n^2} = \lim\limits_{n\to\infty} \dfrac{1}{1+\frac{2}{n}} \cdot \dfrac{-2}{\cancel{n^2}} \cdot \cancel{n^2} = \dfrac{1}{1+0} \cdot (-2) = -2$

(b) $\left\{\frac{(-1)^n n!}{4^n}\right\}$

$\boxed{\text{DNE}}$

$a_n = \dfrac{n!}{4^n}$ diverges since when $n \geq 8$

$a_{n+1} = \dfrac{(n+1)!}{4^{n+1}} = \dfrac{n+1}{4} \cdot \dfrac{n!}{4^n} > 2 \cdot a_n$

Since the sequence is alternating and $\frac{n!}{4^n}$ grows without bound, the sequence diverges

(c) $\left\{\frac{\ln\left(\frac{1}{n}\right)}{n^2}\right\}$

$\boxed{0}$

use L'Hop "$\frac{\infty}{\infty}$"

$\lim\limits_{n\to\infty} \dfrac{\ln(1/n)}{n^2} = \lim\limits_{n\to\infty} \dfrac{\frac{1}{1/n} \cdot \frac{-1}{n^2}}{2n} = \lim\limits_{n\to\infty} \dfrac{-n/n^2}{2n}$

$= \lim\limits_{n\to\infty} \dfrac{-1}{n} \cdot \dfrac{1}{2n} = \lim\limits_{n\to\infty} \dfrac{-1}{2n^2} = 0$