Math 1552 Summer 2023
Quiz 5 *QUP only* July 6
Due date: Sunday at 11:59PM

Name (Print): Key

Teaching Assistant/Section:

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: Sal

For Question (0.) below please list any **outside resources** you used to help solve quiz problems. You can use calculators, texbook/course documents, websites, solving tools, or each other (e.g., TI-89 calculator, textbook formula sheet on page 281, 3Blue1Brown YouTube video on integrals, WolframAlpha, Symbolab). **Be specific.** List the name of anyone who helped you. If you used no outside resources, write `N/A`.

As always, anything you submit must be your own work. Never submit the work of someone else.

Please clearly organize your work, show all steps, simplify all answers, and $\boxed{\text{BOX}}$ your answers.

0. (1 point) *Full credit for accurately following the directions above.*

N/A

1. (3 points) For the given series $\sum a_n$, write the ratio $\frac{a_{n+1}}{a_n}$ from the ratio test. Simplify your answer but *do not* take a limit.

$$\sum_{n=2}^{\infty} \frac{(n-1)!}{(n+1)^2}$$

$$a_{n+1} = \frac{((n+1)-1)!}{((n+1)+1)^2} = \frac{n!}{(n+2)^2}$$

$$\frac{a_{n+1}}{a_n} = \boxed{\frac{n(n+1)^2}{(n+2)^2}}$$

$$a_n = \frac{(n-1)!}{(n+1)^2}$$

$$\frac{a_{n+1}}{a_n} = \frac{n!}{(n+2)^2} \cdot \frac{(n+1)^2}{(n-1)!} = \frac{n!}{(n-1)!} \cdot \frac{(n+1)^2}{(n+2)^2} = \frac{n \cdot (n+1)^2}{(n+2)^2}$$

2. (3 points) Briefly explain the flaw in the following argument. Use complete sentences, justify your reasoning, and use correct terminology from the class.

The series $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ diverges by the direct comparison test, since you can compare the terms $a_n = \frac{n^2}{n^3+1}$ to the terms $b_n = \frac{n^2}{n^3} = \frac{1}{n}$.

Note there is a third page to the quiz this week.

$a_n = \frac{n^2}{n^3+1} \ngeq \frac{n^2}{n^3} = b_n$ since for example

when $n=2$ $a_2 = \frac{4}{9}$ and $b_2 = \frac{1}{2}$ and $\frac{4}{9} \ngeq \frac{1}{2}$.

Since $a_n \ngeq b_n$, even though $\sum b_n$ diverges (p-series w/ $p=1$)

we can <u>NOT</u> conclude that $\sum a_n$ diverges.

3. (14 points) Determine if each series converges or diverges. Fully justify your answer for credit, *e.g.*, state the convergence test you used and clearly state the necessary conditions for the test you are using. Points will be deducted for arguments that are not clearly organized.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ limit comparison w/ $b_n = 1/n$.

$$a_n = \frac{n^2}{n^3+1} \qquad b_n = \frac{1}{n}$$

$$\frac{a_n}{b_n} = \frac{n^2}{n^3+1} \cdot \frac{n}{1} = \frac{n^3}{n^3+1} \longrightarrow 1 = C$$

Since $C > 0$, by limit comparison either $\sum a_n$ & $\sum b_n$ both converge or both diverge.

Finally, since $\sum b_n$ diverges by p-series test w/ $p = 1 \leq 1$.

The series $\boxed{\sum a_n \text{ also diverges}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{1}{3n}\right)^{n^2}$

<u>root test</u>

$$a_n = \left(1 - \frac{1}{3n}\right)^{n^2}$$

$$(a_n)^{1/n} = \left(\left(1 - \frac{1}{3n}\right)^{n^2}\right)^{1/n} = \left(1 - \frac{1}{3n}\right)^n$$

Since $\displaystyle\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$

we have that

$$\lim_{n\to\infty} (a_n)^{1/n} = e^{-1/3} = \frac{1}{e^{1/3}} = L < 1$$

(since $e^{1/3} > 1$)

Since $L < 1$, by the root test

$\boxed{\sum a_n \text{ converges}}$