Math 1552 Summer 2023
Quiz 6 *QUP only* July 13
Due date: Sunday at 11:59PM

Name (Print): Key

Teaching Assistant/Section:

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: Sal

For Question (0.) below please list any **outside resources** you used to help solve quiz problems. You can use calculators, **each other**, texbook/course documents, websites, solving tools (e.g., TI-89 calculator, textbook formula sheet on page 281, 3Blue1Brown YouTube video on integrals, WolframAlpha, Symbolab). **Be specific**, and include the **name** of anyone you discussed quiz problems with. If you used no outside resources, write `N/A`.

As always, anything you submit must be your own work. Never submit the work of someone else.

Please clearly organize your work, show all steps, simplify all answers, and $\boxed{\text{BOX}}$ your answers.

0. (1 point) *Full credit for accurately following the directions above.*

N/A

1. (4 points) Find the interval I and radius R of convergence of the given power series. For the interval of convergence, give your answer using interval notation or using inequality notation.

$$\sum_{n=1}^{\infty} \frac{x^n}{2^n\sqrt{n}}$$

$$\frac{a_{n+1}}{a_n} = \frac{x^{\cancel{n+1}}}{2^{\cancel{n+1}}\cdot\sqrt{n+1}} \cdot \frac{\cancel{2^n}\cdot\sqrt{n}}{\cancel{x^n}} = \sqrt{\frac{n}{n+1}}\cdot\frac{x}{2}$$

$I = \boxed{[-2, 2)}$

$R = \boxed{2}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \sqrt{1}\cdot\frac{x}{2} = L$$

If $|L| < 1$ then $-1 < \frac{x}{2} < 1 \iff -2 < x < 2$

& need to check $x = \pm 2$ separately

@ $x = 2$

$$\sum \frac{2^n}{2^n\sqrt{n}} = \sum \frac{1}{\sqrt{n}}$$

diverges by p-test w/ $p = 1/2 < 1$

@ $x = -2$

$$\sum \frac{(-2)^n}{2^n\cdot\sqrt{n}} = \sum (-1)^n \frac{1}{\sqrt{n}}$$

since $\frac{1}{\sqrt{n}} \to 0$

converges by alt. series test

2. (5 points) Find the Taylor series expansion of $f(x)$ at $x=0$ for the given function. If you use a known (common) Taylor series, please carefully state the known series that you are using as part of your work.

$f(x) = \dfrac{2x}{9+x^2}$

Note there is a third page to the quiz this week.

We know that (★ b/c common Taylor series)

$$\frac{1}{1-x} \stackrel{\star}{=} \sum_{k=0}^{\infty} x^k \quad (\text{if } |x|<1)$$

$$f(x) = \boxed{\sum_{k=0}^{\infty} \frac{2 \cdot x^{2n+1}}{9^{n+1}}}$$

So

$$\frac{2x}{9+x^2} = \frac{2x}{9} \cdot \frac{1}{1+\frac{x^2}{9}} = \frac{2x}{9} \cdot \frac{1}{1-\left(-\frac{x^2}{9}\right)}$$

$$\stackrel{\star}{=} \frac{2x}{9} \cdot \sum_{k=0}^{\infty} \left[\frac{-x^2}{9}\right]^n$$

$$= \frac{2x}{9} \cdot \sum_{k=0}^{\infty} \frac{(-1)^n x^{2n}}{9^n} = \sum_{k=0}^{\infty} \frac{2 \cdot}{9^{n+1}} x^{2n+1}$$

3. (10 points) Determine if the given alternating series converges absolutely, converges conditionally, or diverges.

(a) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{n-1}}$

$\boxed{\text{Converges absolutely}}$

$a_n = \frac{1}{n\sqrt{n-1}} \xrightarrow[n\to\infty]{} 0$ so $\sum (-1)^n a_n$ converges by alt. series test.

Does $\sum a_n$ converge also? Try limit comparison w/ $b_n = \frac{1}{n\sqrt{n}}$

$\frac{a_n}{b_n} = \frac{1}{\cancel{n}\sqrt{n-1}} \cdot \frac{\cancel{n}\sqrt{n}}{1} = \sqrt{\frac{n}{n-1}}$

So $\lim_{n\to\infty} \frac{a_n}{b_n} = \sqrt{1} = 1 = c$

Since $c > 0$ and $\sum b_n$ converges by p-test w/ $p = 3/2 > 1$, the series $\sum a_n$ also converges

(b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

$\boxed{\text{Converges conditionally}}$

$a_n = \frac{1}{n\ln n} \to 0$ as $n \to \infty$ so

so $\sum (-1)^n a_n$ converges.

$\sum a_n$ converges too? Try integral test

$\int_2^N \frac{1}{x\ln x}\,dx = \int_*^* \frac{1}{u}\,du = \ln|u| \Big|_*^*$

$= \ln|\ln x| \Big|_2^N = \ln(\ln N) - \ln(\ln 2)$

Since $\lim_{N\to\infty} \ln(\ln N) = +\infty$ DNE, the series $\sum a_n$ diverges by the integral test