Math 1552 Summer 2023
Quiz 6 July 13
Time limit: 20 Minutes

Name (Print): Key

Teaching Assistant/Section:

GT ID:

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Please clearly organize your work, show all steps, simplify all answers, and $\boxed{\text{BOX}}$ your answers.

1. (5 points) Find the interval I and radius R of convergence of the given power series. For the interval of convergence, give your answer using interval notation or using inequality notation.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}x^n}{3^n}$$

$a_{n+1} = \dfrac{\sqrt{n+1}\, x^{n+1}}{3^{n+1}}$

$a_n = \dfrac{\sqrt{n}\, x^n}{3^n}$

$\dfrac{a_{n+1}}{a_n} = \dfrac{\sqrt{n+1}\, \cancel{x^{n+1}}}{\cancel{3^{n+1}}} \cdot \dfrac{\cancel{3^n}}{\sqrt{n}\, \cancel{x^n}} = \sqrt{\dfrac{n+1}{n}} \cdot \dfrac{x}{3}$

$\lim\limits_{n\to\infty} \dfrac{a_{n+1}}{a_n} = \sqrt{1} \cdot \dfrac{x}{3} = \dfrac{x}{3} = L$

$|L| < 1 \Rightarrow -1 < \frac{x}{3} < 1 \Rightarrow -3 < x < 3$

test $x = \pm 3$ separately

$\sum \frac{\sqrt{n}}{3^n}(-3)^n = \sum (-1)^n \sqrt{n}$ $\qquad \sum \frac{\sqrt{n}}{3^n}(3)^n = \sum \sqrt{n}$

diverges by divergence test

$I = \boxed{(-3, 3)}$

$R = \boxed{3}$

2. (5 points) Find the Taylor series expansion of $f(x)$ at $x = 0$ for the given function. If you use a known (common) Taylor series, please carefully state the known series that you are using as part of your work.

$$f(x) = \frac{4x}{1 + x^3}$$

we know that $\dfrac{1}{1-x} = \displaystyle\sum_{k=0}^{\infty} x^k$ (if $|x| < 1$)

so $\dfrac{4x}{1+x^3} = 4x \cdot \dfrac{1}{1-(-x^3)} = 4x \cdot \displaystyle\sum_{k=0}^{\infty} (-x^3)^n$

$= 4x \displaystyle\sum_{k=0}^{\infty} (-1)^n \cdot x^{3n} = \sum_{k=0}^{\infty} (-1)^n \cdot 4 \cdot x^{3n+1}$

$f(x) = \boxed{\displaystyle\sum_{k=0}^{\infty} (-1)^n \cdot 4 \cdot x^{3n+1}}$

3. (10 points) Determine if the given alternating series converges absolutely, converges conditionally, or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^3+1}}$

$\boxed{\text{converges absolutely}}$

$a_n = \frac{1}{\sqrt{n^3+1}} \to 0$ as $n \to \infty$

So series $\sum (-1)^n a_n$ converges. by alt. series test

$\sum a_n$ converges? Try limit comparison w/ $b_n = \frac{1}{n^{3/2}}$

$\frac{a_n}{b_n} = \frac{1}{\sqrt{n^3+1}} \cdot \frac{n^{3/2}}{1} = \sqrt{\frac{n^3}{n^3+1}} \xrightarrow[n\to\infty]{} 1 = c$

Since $c > 0$ and $\sum b_n$ converges by p-test w/ $p = 3/2$, The series $\sum a_n$ also converges

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n!}{3^n}$

$\boxed{\text{diverges}}$

$a_n = \frac{n!}{3^n}$, notice $a_{n+1} = \frac{(n+1)!}{3^{n+1}} = \frac{n+1}{3} \cdot \frac{n!}{3^n} = \frac{n+1}{3} \cdot a_n$

Since $\lim_{n\to\infty} a_n = +\infty$ DNE

The series $\sum (-1)^n a_n$ diverges by the divergence test.