Math 1552 Summer 2023 **Name (Print):** Key

Quiz 6 Practice July 13

Teaching Assistant/Section:

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Please clearly organize your work, show all steps, simplify all answers, and $\boxed{\text{BOX}}$ your answers.

1. (3 points) Find the interval I and radius R of convergence of the given power series. For the interval of convergence, give your answer using interval notation or using inequality notation.

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt{n}}$$

Ratio test

$a_{n+1} = \frac{(-1)^{n+1} x^{n+1}}{\sqrt{n+1}}$

$a_n = \frac{(-1)^n \cdot x^n}{\sqrt{n}}$

If $|L| = 1$ then $-1 < x < 1$

test $x = \pm 1$ individually

@ $x=1$ $\sum \frac{(-1)^n 1^n}{\sqrt{n}}$ converges by alt series test since $\frac{1}{\sqrt{n}} \to 0$ as $n \to \infty$

$I = \boxed{(-1, 1]}$

$R = \boxed{1}$

$$\frac{a_{n+1}}{a_n} = \frac{\cancel{(-1)^{n+1}}\cancel{x^{n+1}}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{\cancel{(-1)^n} \cdot \cancel{x^n}} = \sqrt{\frac{n}{n+1}} \cdot -x = L$$

@ $x = -1$ $\sum \frac{(-1)^n(-1)^n}{\sqrt{n}} = \sum \frac{1}{\sqrt{n}}$ diverges by p-series test w/ $p = 1/2 < 1$

2. (3 points) Find the Taylor series expansion of $f(x)$ at $x = 0$ for the given function. If you use a known (common) Taylor series, please carefully state the known series that you are using as part of your work.

$f(x) = x^2 e^{3x}$

We know that (* by common Taylor series)

$$e^x \overset{*}{=} \sum_{k=0}^{\infty} \frac{1}{k!} x^k \quad \text{(for all } x\text{)}$$

$$f(x) = \boxed{\sum_{k=0}^{\infty} \frac{3^k}{k!} x^{k+2}}$$

So

$$x^2 e^{3x} \overset{*}{=} x^2 \sum_{k=0}^{\infty} \frac{1}{k!}(3x)^k = \sum_{k=0}^{\infty} \frac{1}{k!} x^2 \cdot 3^k \cdot x^k$$

$$= \sum_{k=0}^{\infty} \frac{3^k}{k!} x^{k+2}$$

3. (6 points) Determine if the given alternating series converges absolutely, converges conditionally, or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}$

$\boxed{\text{Converges conditionally}}$

$a_n = \frac{1}{\sqrt{n^2+1}} \xrightarrow[n\to\infty]{} 0$ so $\sum (-1)^n a_n$ converges

by alternating series test.

Does $\sum a_n$ also converge? use limit comparison w/ $b_n = 1/n$

$\frac{a_n}{b_n} = \frac{1}{\sqrt{n^2+1}} \cdot \frac{n}{1} = \sqrt{\frac{n^2}{n^2+1}}$

So $\lim_{n\to\infty} \frac{a_n}{b_n} = \sqrt{1} = 1 = c$

Since $c>0$ and $\sum b_n$ diverges by p-series w/ $p=1 \leq 1$, the series $\sum a_n$ also diverges

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 3n}{\ln n}$

$\boxed{\text{diverges}}$

$a_n = \frac{3n}{\ln n}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3}{1/n} = \lim_{n\to\infty} 3n = +\infty$ DNE

L'Hop $\frac{\infty}{\infty}$

So $\sum (-1)^n a_n$ diverges by the divergence test.