Math 1553 **Exam 1** Spring 2016

Instructor: Sal Barone

Name: KEY

GT username: ____________

1. No books or notes are allowed.
2. All calculators and/or electronic devices are not allowed.
3. Show all work and fully justify your answer to receive full credit. *Hint: on most problems it is easy to check your answers!*
4. Please $\boxed{\text{BOX}}$ your answers.
5. Good luck!

Page	Max. Possible	Points
1	30	
2	30	
3	40	
Total	100	

1. Solve the system of linear equations, or state that the system is inconsistent. (10 pts.)

$$\begin{aligned} x_1 + 2x_2 \ +3x_3 - 2x_4 &= 1 \\ -3x_1 - 6x_2 \ -9x_3 + 7x_4 &= 0 \\ -2x_1 - 4x_2 \ -6x_3 + 5x_4 &= 1 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & -2 & 1 \\ -3 & -6 & -9 & 7 & 0 \\ -2 & -4 & -6 & 5 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & -2 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 7 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\boxed{\begin{aligned} x_1 &= -2x_2 - 3x_3 + 7 \\ x_2 &= \text{free} \\ x_3 &= \text{free} \\ x_4 &= +3 \end{aligned}}$$

(a) Check your answer for the above problem. (5 pts.)

$x_1 = 7$
$x_2 = 0$
$x_3 = 0$
$x_4 = +3$

EQN 1 $7 + 0 + 0 - 2(+3) \stackrel{?}{=} 1$ ✓
EQN 2 $-3 \cdot 7 + 0 + 0 + 7(+3) = 0$ ✓
EQN 3 $-2(7) + 0 + 0 + 5(+3) = 1$ ✓

2. Express the vector $\begin{bmatrix} -5 \\ -2 \\ -1 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 9 \\ 0 \\ 9 \end{bmatrix}$, $\begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}$. (15 pts.)

$$\left[\begin{array}{ccc|c} 9 & 6 & -3 & -5 \\ 0 & 3 & 0 & -2 \\ 9 & 6 & 0 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 9 & 6 & -3 & -5 \\ 0 & 3 & 0 & -2 \\ 0 & 0 & 3 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 9 & 0 & -3 & -1 \\ 0 & 1 & 0 & -2/3 \\ 0 & 0 & 3 & 4 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 9 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2/3 \\ 0 & 0 & 1 & 4/3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & -2/3 \\ 0 & 0 & 1 & 4/3 \end{array}\right].$$

$$\boxed{\begin{bmatrix} -5 \\ -2 \\ -1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 \\ 0 \\ 9 \end{bmatrix} - \frac{2}{3}\begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix} + \frac{4}{3}\begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}}$$

check: $\frac{1}{3}(9) - \frac{2}{3}(6) + \frac{4}{3}(-3) =$
$3 - 4 - 4 = -5$ ✓
$0 + -2/3(3) + 0 = -2$ ✓
[illegible] ✓

ok

3. Are the vectors linearly independent? Justify your answer fully for full credit. (15 pts.)

$$\left\{ \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$$

$$\begin{bmatrix} -2 & 2 & 1 \\ 2 & -1 & -1 \\ -3 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} -2 & 2 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \sim \cdots \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\boxed{\begin{array}{c} \text{linearly independent!} \\ \text{Since } Ax=0 \text{ has only trivial solution.} \end{array}}$$

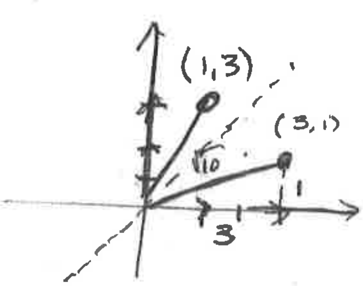
4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first makes a horizontal shear where e_2 maps to $e_2 + 3e_1$ (e_1 remains unchanged), and then reflects about the line $x_1 = x_2$. Find the standard matrix A of T. Partial credit will be given if the standard matrix of either the first part, the shear, or the second part, the reflection, are found correctly. (15 pts.)

$T = R \circ S$

$$\boxed{T = \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix}}$$

$S = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$R = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$



5. In the questions below, assume that A is a (general) $m \times n$ matrix, with m rows and n columns, and b is a (general) vector in $\mathbb{R}^m$. Circle TRUE if the statement is **always** true (for any possible choice of A and b), otherwise circle FALSE. (4 pts. each)

(a) If $m > n$, then $Ax = b$ has a unique solution. TRUE FALSE

$[A|b] \sim \left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$ inconsistent $\Rightarrow$ no soln

(b) If $m = n$, then $Ax = b$ has a unique solution. TRUE FALSE

$[A|b] \sim \left[\begin{array}{cc|c} 1 & 8 & 2 \\ 0 & 0 & -1 \end{array}\right]$ inconsistent $\Rightarrow$ no soln.

(c) If $Ax = b$ has a unique solution, then there are n pivots in A. TRUE FALSE

(d) If there are n pivots in A, then $Ax = b$ has a unique solution. TRUE FALSE

See (a)

(e) If $Ax = 0$ has more than one solution, then it has infinitely many solutions. TRUE FALSE

needs to be 0.

(f) The matrix $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is in rref. TRUE FALSE

(g) The matrix $\begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in rref. TRUE FALSE

(h) If $Ax = b$ has a unique solution, then b is in the span of the columns of A. TRUE FALSE

(i) If $m > n$, then the columns of A are linearly independent. TRUE FALSE

$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$ $\begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 3 & 12 \end{bmatrix}$

(j) If $m < n$, then the columns of A are linearly dependent. TRUE FALSE

$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}$ must have pivot

$\Rightarrow Ax = 0$ has ∞-many solns.

$\Rightarrow$ cols. of A lin dependent.