Math 1553 **Exam 1** Spring 2016

Instructor: Sal Barone (B)

Name: KEY

GT username: ______________________

1. No books or notes are allowed.
2. All calculators and/or electronic devices are not allowed.
3. Show all work and fully justify your answer to receive full credit. *Hint: on most problems it is easy to check your answers!*
4. Please $\boxed{\text{BOX}}$ your answers.
5. Good luck!

Page	Max. Possible	Points
1	30	
2	30	
3	40	
Total	100	

1. Solve the system of linear equations, or state that the system is inconsistent. (10 pts.)

$$\begin{aligned} 2x_1 - 2x_2 \quad -x_3 - 2x_4 &= -1 \\ -6x_1 + 6x_2 \quad +5x_3 + 10x_4 &= -3 \\ 3x_1 - 3x_2 \quad -2x_3 - 4x_4 &= 0 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 2 & -2 & -1 & -2 & -1 \\ -6 & 6 & 5 & 10 & -3 \\ 3 & -3 & -2 & -4 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 2 & -2 & -1 & -2 & -1 \\ 0 & 0 & 2 & 4 & -6 \\ 3 & -3 & -2 & -4 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 2 & -2 & -1 & -2 & -1 \\ 0 & 0 & 1 & 2 & -3 \\ 3 & -3 & 0 & 0 & -6 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 2 & -3 \\ 2 & -2 & -1 & -2 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\boxed{\begin{aligned} x_1 &= -2 + x_2 \\ x_2 &= \text{free} \\ x_3 &= -3 - 2x_4 \\ x_4 &= \text{free} \end{aligned}}$$

(a) Check your answer for the above problem. (5 pts.)

If $x_2 = 0, x_4 = 0$

$x_1 = -2$
$x_2 = 0$
$x_3 = -3$
$x_4 = 0$

$$\boxed{\begin{aligned} 2(-2) - 0 - (-3) + 0 &\stackrel{?}{=} -1 \; \checkmark \\ -6(-2) - 0 + 5(-3) + 0 &\stackrel{?}{=} -3 \; \checkmark \\ 3(-2) - 0 - 2(-3) + 0 &\stackrel{?}{=} 0 \; \checkmark \end{aligned}}$$

2. Express the vector $\begin{bmatrix} 4 \\ 3 \\ -4 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} -2 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ (15 pts.)

$$\left[\begin{array}{ccc|c} -2 & 1 & -1 & 4 \\ -2 & -1 & -1 & 3 \\ 4 & 2 & 1 & -4 \end{array}\right] \sim \left[\begin{array}{ccc|c} -2 & 1 & -1 & 4 \\ 0 & -2 & 0 & -1 \\ 0 & 4 & -1 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} -2 & 1 & -1 & 4 \\ 0 & -2 & 0 & -1 \\ 0 & 0 & -1 & 2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & -2 \\ 0 & 1 & 0 & +1/2 \\ 0 & 0 & 1 & -2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 + \frac{1}{4} + (-2)\frac{1}{2} \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & -2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3/4 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\boxed{\left(-\frac{3}{4}\right)\begin{bmatrix} -2 \\ -2 \\ 4 \end{bmatrix} + \left(\frac{1}{2}\right)\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + (-2)\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ -4 \end{bmatrix}}$$

3. Are the vectors linearly independent? Justify your answer fully for full credit. (15 pts.)

$$\left\{ \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 2 & -2 & 3 \\ -2 & 1 & -3 \\ 1 & -1 & 2 \end{bmatrix} \sim \begin{bmatrix} 2 & -2 & 3 \\ 0 & -1 & 0 \\ 1 & -1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 2 & -2 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

linearly independent!

Since $Ax=0$ has only the trivial solution

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first makes a horizontal shear where e_2 maps to $3e_2 + e_1$ (e_1 remains unchanged), and then reflects about the line $x_1 = x_2$. Find the standard matrix A of T. Partial credit will be given if the standard matrix of either the first part, the shear, or the second part, the reflection, are found correctly. (15 pts.)

reflection shear

$T = R \circ S$

$$S(e_1) = e_1$$

$$S(e_2) = 3e_2 + e_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

$$S = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}.$$

$$T = R \circ S$$

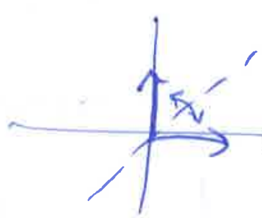
$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$$

$$\boxed{T = \begin{bmatrix} 0 & 3 \\ 1 & 1 \end{bmatrix}}$$

$$R(e_1) = e_2$$

$$R(e_2) = e_1$$

$$R = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



5. In the questions below, assume that A is a (general) $m \times n$ matrix, with m rows and n columns, and b is a (general) vector in $\mathbb{R}^m$. Circle TRUE if the statement is **always** true (for any possible choice of A and b), otherwise circle FALSE. (4 pts. each)

(a) If $Ax = 0$ has more than one solution, then it has infinitely many solutions. TRUE FALSE

(b) If $m = n$, then $Ax = b$ has a unique solution. TRUE FALSE

$$\left[\begin{array}{ccc|c} 1 & 0 & -8 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{array}\right] \rightarrow \text{inconsistent}$$

(c) If $m > n$, then $Ax = b$ has a unique solution. TRUE FALSE

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{array}\right] \Rightarrow \text{inconsistent}$$

(d) If $Ax = b$ has a unique solution, then there are n pivots in A. TRUE FALSE

(e) If there are n pivots in A, then $Ax = b$ has a unique solution. TRUE FALSE

could be inconsistent

(f) If $Ax = b$ has a unique solution, then b is in the span of the columns of A. TRUE FALSE

Should be 0

(g) The matrix $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is in rref. TRUE FALSE

(h) The matrix $\begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in rref. TRUE FALSE

(i) If $m < n$, then the columns of A are linearly dependent. TRUE FALSE

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \circ & \circ \end{bmatrix}$$

(j) If $m > n$, then the columns of A are linearly independent. TRUE FALSE

$$\begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 3 & 12 \end{bmatrix}$$