Math 1553 **Exam 2** Spring 2016

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Name: KEY

GT username:

1. No books or notes are allowed.
2. All calculators and/or electronic devices are not allowed.
3. Show all work and fully justify your answer to receive full credit. *Hint: on most problems it is easy to check your answers!*
4. Please $\boxed{\text{BOX}}$ your answers.
5. Good luck!

Page	Max. Possible	Points
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2	36	
3	25	
4	14	
Total	100	

1. Solve the system below by writing $A\mathrm{x} = b$, find A^{-1} and use it to solve the system for x.

$$\begin{aligned} -x_1 + 3x_2 \quad &= -1 \\ -x_1 - 6x_2 \quad &= \ \ 2 \\ 2x_2 \ \ -x_3 &= -1 \end{aligned}$$

(a) The inverse of A is ... (15 pts.)

$$[A|I] = \left[\begin{array}{ccc|ccc} -1 & 3 & 0 & 1 & 0 & 0 \\ -1 & -6 & 0 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} -1 & 3 & 0 & 1 & 0 & 0 \\ 0 & -9 & 0 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} -1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & +1/9 & -1/9 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} -1 & 0 & 0 & 2/3 & 1/3 & 0 \\ 0 & 1 & 0 & 1/9 & -1/9 & 0 \\ 0 & 0 & -1 & -2/9 & 2/9 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2/3 & -1/3 & 0 \\ 0 & 1 & 0 & 1/9 & -1/9 & 0 \\ 0 & 0 & 1 & 2/9 & -2/9 & -1 \end{array}\right] \qquad \boxed{A^{-1} = \begin{bmatrix} -2/3 & -1/3 & 0 \\ 1/9 & -1/9 & 0 \\ 2/9 & -2/9 & -1 \end{bmatrix}}$$

(b) Using A^{-1} to solve $A\mathrm{x} = b$ we have that x equals ... (10 pts.)

If $Ax = b$ & $b = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$

$\Rightarrow x = A^{-1}b$

$$x = \begin{bmatrix} -2/3 & -1/3 & 0 \\ 1/9 & -1/9 & 0 \\ 2/9 & -2/9 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2/3 - 2/3 \\ -1/9 - 2/9 \\ -2/9 - 4/9 + 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1/3 \\ 1/3 \end{bmatrix}$$

$$\boxed{X = \begin{bmatrix} 0 \\ -1/3 \\ 1/3 \end{bmatrix}}$$

2. Find the **null space** of A and a **basis for the column space** of A.

$$A = \begin{bmatrix} 1 & -4 & -2 \\ 2 & -8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) The null space of A is ... (10 pts.)

$$A = \begin{bmatrix} 1 & -4 & -2 \\ 2 & -8 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & -2 \\ 0 & 0 & 4 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

x_2 free

$$X = x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} \qquad \text{null } A = \boxed{\text{Span}\left\{ \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} \right\}}$$

(b) A basis for the column space of A is ... (10 pts.)

pivots in col 1 & col 3

basis for col A is $\boxed{\left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}}.$

3. Find the determinant of A, and decide whether or not A is invertible.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 3 \\ 1 & 1 & 3 \end{bmatrix} \quad (\text{note } 2\text{col } 1 + \text{col } 2 = \text{col } 3)$$

(a) The determinant $\det(A)$ equals ... (10 pts.)

$$\det(A) = 1 \cdot \begin{vmatrix} 3 & 3 \\ 1 & 3 \end{vmatrix} - 0 + 1 \begin{vmatrix} 0 & 2 \\ 3 & 3 \end{vmatrix}$$

$$= (9-3) + (0-6)$$

$$= 6 - 6 = 0$$

$$\boxed{\det A = 0}$$

(b) The matrix A is INVERTIBLE/$\boxed{\text{NOT-INVERTIBLE}}$ (circle one), and I know this because ... (6 pts.)

$\det A = 0.$

4. Find the standard matrix for the linear transformation T given by the rule (10 pts.)

$$T\left(\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right) = \begin{bmatrix} v_1 - v_2 - 2v_3 \\ v_2 + v_3 \\ v_1 \end{bmatrix}.$$

$T(e_1) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

standard matrix $A = \begin{bmatrix} T(e_1) & T(e_2) & T(e_3) \end{bmatrix}$

$$\boxed{A = \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}}$$

$T(e_2) = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

$T(e_3) = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

5. Find an LU-decomposition for $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 1 & 0 \\ 6 & -3 & 10 \end{bmatrix}$. (15 pts.)

Find U by row reducing

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 1 & 0 \\ 6 & -3 & 10 \end{bmatrix} \sim \begin{matrix} -2R_1 + R_2 \\ -3R_1 + R_3 \end{matrix} \begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

So $\boxed{U = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix} \; \& \; L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}}$

6. In the questions below, assume that A is a (general) $m \times n$ matrix, with m rows and n columns. Circle TRUE if the statement is **always** true (for any possible choice of A), otherwise circle FALSE. (2 pts. each)

(a) If A has n pivot columns, then the columns of A span $\mathbb{R}^n$. TRUE FALSE

(b) If $m = n$, then A is either invertible or has determinant equal to zero. TRUE FALSE

(c) The dimension of span $\left\{ \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -4 \\ -4 \\ 0 \end{bmatrix}, \cancel{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}} \right\}$ is two. ✓ TRUE FALSE

(d) If the linear transformation T_A is one-to-one, then $m = n$. TRUE FALSE

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

(e) The columns of A always span the column space of A. TRUE FALSE

by defn

(f) ~~If A is not invertible, then~~ the rank of A plus the nullity of A is ~~not~~ necessarily n. TRUE FALSE

↑ always true (w/o the "not")

(g) The span of $\left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$. TRUE FALSE

if span omitted then TRUE