1. Let A be the matrix

$$A = \begin{bmatrix} 2 & -2 & 1 & -4 \\ 1 & 5 & -4 & 4 \\ 1 & 2 & 2 & 4 \\ 3 & 6 & -3 & 15 \end{bmatrix}$$

(a) Find a basis for the $\lambda = 3$ eigenspace of A. (12 pts.)

(b) Suppose v_1, v_2 are any $\lambda = 3$ eigenvectors of A that are all distinct, so $Av_i = 3v_i$, for $i = 1, 2$, and $v_i \neq v_j$ if $i \neq j$. Is the set $\{v_1, v_2\}$ a basis for the $\lambda = 3$ eigenspace of A? Justify your answer for full credit. (6 pts.)

2. Find all real or complex eigenvalues of each matrix and an associated eigenvector for each eigenvalue. (10 pts. each)

(a) $\begin{bmatrix} 7 & 3 \\ -2 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 13 \\ -2 & 1 \end{bmatrix}$

3. Find all eigenvalues of the matrix (10 pts.)

$$\begin{bmatrix} 3 & 0 & 3 \\ 0 & 1 & 1 \\ 2 & 0 & 2 \end{bmatrix}$$

4. Suppose A is the non-invertible matrix below which has eigenvalue $\lambda = 6$. Is A diagonalizable? Justify your answer for full credit. (10 pts.)

$$A = \begin{bmatrix} 1 & 3 & 1 & 2 & \text{-1} \\ 1 & 3 & 1 & 2 & \text{-1} \\ 1 & 3 & 1 & 2 & \text{-1} \\ 1 & 3 & 1 & 2 & \text{-1} \\ 1 & 3 & 1 & 2 & \text{-1} \end{bmatrix}$$

5. Suppose A is a 2×2 matrix with $\lambda = 0$ and $\lambda = 1$ as eigenvalues. Choose one: (1) show that $A^2\mathrm{x} = A\mathrm{x}$ for any x in $\mathbb{R}^2$, or (2) show that $A^2 = A$. (8 pts.)

6. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which reflects vectors in $\mathbb{R}^2$ about the line $y = x$. Find eigenvectors and state the associated eigenvalues for the standard matrix A of this linear transformation. *Hint: think geometrically.* (8 pts.)

7. Find a basis for the $\lambda = a+b$ and the $\lambda = a-b$ eigenspaces of the matrix $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$, assuming neither a nor b is zero. (8 pts.)

8. Let A be the matrix below. Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. Assume none of a, b, c, d, e are zero. *Hint: think geometrically and consider the previous problem.* (8 pts.)

$$\begin{bmatrix} a & b & 0 & 0 & 0 \\ b & a & 0 & 0 & 0 \\ 0 & 0 & c & 0 & 0 \\ 0 & 0 & 0 & d & e \\ 0 & 0 & 0 & e & d \end{bmatrix}$$

9. Let A be a 2×2 matrix which satisfies $A\mathbf{v}_1 = 3\mathbf{v}_1$ and $A\mathbf{v}_2 = -\mathbf{v}_2$, where $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$. If $\mathbf{x}$ is the vector $\mathbf{x} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$, compute $A^3\mathbf{x}$. (10 pts.)