Handwritten Homework Assignments - Exploration for MATH 1554
For each assignment, complete the questions by hand on paper/tablet. Write neatly and use complete sentences where necessary. You must submit original work, but y'all can share ideas. Handwritten homework is due on Sunday in Gradescope; no late submissions accepted.

Week 1: Practice sketching in $2 D$ and $3 D$. In $\mathbb{R}^{2}$, using the coordinates $x_{1}, x_{2}$, sketch the following making a new sketch for each part (a)-(b):
(a) sketch two random lines in $\mathbb{R}^{2}$ and label each line with the equation describing it, and draw and label the point of intersection and at least one other point on each line. Verify with a calculation that each point really belongs to each line it is drawn on.
(b) sketch one random line and a parallel line and label each line with the equation describing it, and again verify with a calculation that each point really is on the line you say it is on.

In $\mathbb{R}^{3}$, using the coordinates $x_{1}, x_{2}, x_{3}$, sketch the following making a new sketch for each part (i)-(v):
(i) use the exact same equations from part (a) above in $\mathbb{R}^{2}$ above, but with the variables $x_{1}, x_{2}, x_{3}$ and understanding that the coefficient of $x_{3}$ should be understood to be zero; sketch the two planes and the line of intersection; for each plane/line draw at least two points with different $x_{3}$-values; check with calculations that your sketched points are on each object that it belongs on;
(ii) sketch the plane consisting of points satisfying the condition that $x_{1}=0$ (the back wall), and sketch at least three non-colinear points;
(iii) sketch the plane $x_{3}=0$ and the plane $x_{1}=0$ on the same axes, and sketch and label the line of intersection of these two planes;
(iv) sketch a line which passes through the origin which is not contained in any of the three coordinate planes, include and label at least three points on the line, you do not have to label the line;
(v) sketch the plane defined by $3 x+y+2 z=6$, include and label at least four points in this plane no three of which are colinear, and at least one of the points should not lie in any of the coordinate axes or coordinate planes.

Main idea: In each problem, you are practicing drawing an accurate, representative graph of the plane (or line) of points which satisfy the given equation in the variables $x_{1}, x_{2}$, and $x_{3}$, and including a few points on the geometric object (plane/line), and verifying with a calculation that your point really belongs to the object on which it has been drawn.

Week 2: Practice with span, linear combination, and inconsistent systems. (a) Choose two vectors $\vec{v}, \vec{w}$ in $\mathbb{R}^{2}$ and a third vector $\vec{b}$ also in $\mathbb{R}^{2}$, and express $b$ as a linear combination of $v, w$ by finding scalars $c_{1}, c_{2}$ such that $c_{1} \vec{v}+c_{2} \vec{w}=\vec{b}$. Sketch the situation in $\mathbb{R}^{2}$ with an illustration that uses the parallelogram rule and verify with calculations that $\vec{b}$ is really a linear combination of $\vec{v}$ and $\vec{w}$. (b) Repeat part (a) with new vectors in $\mathbb{R}^{3}$ such that the augmented matrix $[\vec{v} \vec{w} \mid \vec{b}]$ gives a consistent system, again illustrating by graphing but this time in $\mathbb{R}^{3}$. (c) Why is it harder to find a consistent system for part (b) compared to part (a)? Explain your idea $\left(^{*}\right.$ ) clearly using complete sentences. Note(*): we are looking for something different than "it is harder to draw things in three dimensions" here - try to explain why finding a consistent system is harder for part 2 compared to part 1.

## Warning!

PLEASE NOTE: Your submissions for this and future exploration assignments need to contain vectors which are general looking. Choosing vectors which are scalar multiples of $\left[\begin{array}{l}1 \\ 0\end{array}\right],\left[\begin{array}{l}0 \\ 1\end{array}\right]$, or $\left[\begin{array}{l}1 \\ 0 \\ 0\end{array}\right],\left[\begin{array}{l}0 \\ 1 \\ 0\end{array}\right],\left[\begin{array}{l}0 \\ 0 \\ 1\end{array}\right]$, or have too many zeros or ones, or are otherwise too simple and miss the point of the exploration will receive a deduction of points.

Please, do not ask on Piazza if your vectors are general enough to get full credit. The explorations are assignments which require you to make a judgement call, to explore a particular concept of the course and NOT to come up with the simplest example which satisfies the minimum requirements of the assignment.

Week 3: Learn some basics of MATLAB. See Sal's personal website for links to the MATLAB \#1 Exploration. (requires MATLAB installation or MathWorks access to MATLAB online)

Week 4: Propositional logic. This week we will explore the basics of propositional logic in order to help develop some framework to practice True/False questions when studying for the exam. The assignment this week is to complete the three questions Q1, Q2, Q3 and upload to Gradescope. The additional (non-question) text is just to help you understand the assignment.

Let's start with the main definitions:

Definition: A mathematical statement (aka, a proposition) is a sentence which is either true or false.

Each of the following are statements; they can be true or false, depending on the choice of vectors and/or matrices in each statement.

1. $A \vec{x}=\vec{b}$ is consistent.
2. $[A \mid \vec{b}]$ is row equivalent to $[C \mid \vec{d}]$.
3. $A \vec{x}=\overrightarrow{0}$ has a non-trivial solution.
4. The columns of $A$ are linearly dependent.
5. $T(\vec{x})=A \vec{x}$ is onto.

For example, the first statement (1.) is true if $A=\left[\begin{array}{ll}1 & 0 \\ 0 & 1 \\ 0 & 0\end{array}\right]$ and $\vec{b}=\left[\begin{array}{l}1 \\ 2 \\ 0\end{array}\right]$, but this statement is false if $A=\left[\begin{array}{ll}1 & 0 \\ 0 & 1 \\ 0 & 0\end{array}\right]$ and $\vec{b}=\left[\begin{array}{l}1 \\ 2 \\ 3\end{array}\right]$.

Q1: You try it! Pick one of the other statements (2.)-(5.) above and come up with choices for the vectors/matrices that make the statement true, and choices that make the same statement false. Check with calculations that your example works for each. Note: please write the problem statement of the problem you are solving, to help the grader.

Statements can be combined in a variety of ways to create new statements, using for example AND, OR, or IMPLIES.
For example, each of the following are statements. (connecting word highlighted for emphasis)
6. $[A \mid \vec{b}]$ is row equivalent to $[C \mid \vec{d}]$ and $A \vec{x}=\vec{b}$ is consistent.
7. $A \vec{x}=\overrightarrow{0}$ has a non-trivial solution or $T(\vec{x})=A \vec{x}$ is onto.

8a. The columns of $A$ are linearly dependent implies $A \vec{x}=\overrightarrow{0}$ has a non-trivial solution.
8b. If the columns of $A$ are linearly dependent, then $A \vec{x}=\overrightarrow{0}$ has a non-trivial solution.
The most common way that an IMPLIES statement is written is to use if and then. The statements (8a.) and (8b.) say exactly the same thing using different wording.

An implication statement is true whenever knowing that the "if part" is true forces the "then part" to also be true. So (8a.) is a true implication because whenever the columns of $A$ are linearly dependent there is a free variable in the system $A \vec{x}=\overrightarrow{0}$, and assigning a non-zero value to the free variable gives a non-zero $\vec{x}$ which satisfies $A \vec{x}=\overrightarrow{0}$.

On the other hand, an implication statement is false if for some choice making the "if part" true, the "then part" is false. For example consider the following false implication.
9. If $A \vec{x}=\vec{b}$ and $A \vec{y}=\vec{b}$, then $A(\vec{x}+\vec{y})=\vec{b}$.

This implication is false because there is a counterexample for which the first part is true, but the second part is false. For example, choosing $A=\left[\begin{array}{ll}1 & 1 \\ 1 & 1\end{array}\right]$ and $\vec{b}=\left[\begin{array}{l}3 \\ 3\end{array}\right]$ we see that $\vec{x}=\left[\begin{array}{l}1 \\ 2\end{array}\right]$ and $\vec{y}=\left[\begin{array}{l}3 \\ 0\end{array}\right]$ both satisfy $A \vec{x}=\vec{b}$ and $A \vec{y}=\vec{b}$, but $\vec{x}+\vec{y}=\left[\begin{array}{l}4 \\ 2\end{array}\right]$ and so $A(\vec{x}+\vec{y})=\left[\begin{array}{l}6 \\ 6\end{array}\right]$ (and $\left[\begin{array}{l}6 \\ 6\end{array}\right]$ is not $\vec{b}$ ).

Q2: You try it! Select any one TRUE and any one FALSE true/false question from any of the practice exams that use IMPLIES (aka an if-then statement). For each of the two problems, identify the propositions in the problem (the "if part" and the "then part"). If the implication is false, provide a counter-example with explanation/calculations to show why it works. If the implication is true, give a short general* explanation using precise and correct terminology from class. (* a short general proof - not an example) Note: please write the problem statement of the problem you are solving, to help the grader.

Finally, some statements can have one or more mathematical quantifiers. There are two kinds of mathematical quantifiers, which are the universal quantifier for all (aka for every), and the existential quantifier for some (aka there exists).
For example, each of the following statements has a quantifier. (quantifier highlighted)
10. If $A \vec{x}=\vec{b}$ is consistent for some $\vec{b} \in R^{m}$, then $A$ has a pivot in every row.
11. If $A$ does not have a pivot in every column and $T(\vec{x})=A \vec{x}$, then for every $\vec{x}_{1}$ and $\vec{x}_{2}$ with $\vec{x}_{1} \neq \vec{x}_{2}$ we have that $T\left(\vec{x}_{1}\right)=T\left(\vec{x}_{2}\right)$.

Q3: You try it! In (10.) and (11.) identify the propositions in each problem. The implications (10.) and (11.) are both false; provide a counterexample to one of the implications. Give a short description (one or two short sentences) as to why your counterexample works, and verify any assertions you make with calculations.
Note: please write the problem statement of the problem you are solving, to help the grader.

Hint: For (10.) a counterexample will be a matrix $A$ and a vector $\vec{b}$ such that $A \vec{x}=\vec{b}$ is consistent, but $A$ does not have a pivot in every row.

Hint: For (11.) a counterexample will be a matrix $A$ that does not have a pivot in every column, and such that there exist vectors $\vec{x}_{1}$ and $\vec{x}_{1}$ such that $\vec{x}_{1} \neq \vec{x}_{2}$ and $T\left(\vec{x}_{1}\right) \neq T\left(\vec{x}_{2}\right)$.

Week 5: More true/false practice. Consider the following problems from Exam 1.

1. 1 (a) iii T/F (Spring '23)

If $\vec{v}$ and $\vec{w}$ are solutions to an inhomogeneous system $A \vec{x}=\vec{b}$, then $\vec{v}-\vec{w}$ is a solution to $A \vec{x}=\overrightarrow{0}$.
2. 1(a)v T/F (Spring ' ${ }^{23 \text { ) }}$

If $A$ is size $3 \times 4$ and none of the rows of $A$ consist entirely of zeros, then $A$ has 3 pivots.

## 3. 1(b)i possible/impossible (Spring '23)

An $m \times n$ matrix $A$ with a pivot in its last column such that $A \vec{x}=\overrightarrow{0}$ is inconsistent.
4. 1(b)i T/F - modified (Spring '23) Note: now $T / F$ and inhomogeneous.

If $A$ is an $m \times n$ matrix with a pivot in its last column, then $A \vec{x}=\vec{b}$ is inconsistent for any choice of vector $\vec{b}$.
5. 1(b)ii possible/impossible (Spring '23)

Two nonzero vectors $\vec{v}_{1}, \vec{v}_{2}$ such that $\left\{\vec{v}_{1}, \vec{v}_{2}\right\}$ is linearly independent and $\left\{\vec{v}_{1}-\vec{v}_{2}, \vec{v}_{1}+\vec{v}_{2}\right\}$ is linearly dependent.

For each of the true/false and possible/impossible problems above do all of the following.
(i) Write the problem and provide the answer.
(ii) (*)Complete RULE \#1: do one example which either demonstrates that the statement is satisfied (an example which shows that the statement could be true) or that the statement is not satisfied (a counter-example which shows that the statement must be false). For example, for (1.) you would need to find a matrix $A$, and vectors $\vec{v}, \vec{w}, \vec{b}$ so that $\vec{v}$ and $\vec{w}$ are solutions to $A \vec{x}=\vec{b}$ and then check if $\vec{v}-\vec{w}$ is a solution to $A \vec{x}=\overrightarrow{0}$ or not. For your example, say whether your example was a counter-example or not.
(iii) Identify the relevant definitions in the statement. For example, for (2.) you need to define (a) size of a matrix, and (b) what a pivot is. You must use the precise definition from the textbook/lecture, here. Do not summarize or provide some intuitive definition! I want the textbook definition.
$\left(^{*}\right)$ Note: For part (ii), your example does not have to be a counter-example if the statement is false. Just do any example.
Finally, create a NEW Problem (6.) that you make by picking one of the five problems and modifying it in some way. For example, you can change the premise of an implication or its conclusion. Or you can make a true/false into an impossible/possible, or visa versa. Another idea would be to change the order of the implication by swapping the if-then statements, or negating either the if-part, or the then-part, or both.

Repeat steps (i)-(ii) for problem (6.) and indicate which of the 5 problems you are modifying.

Week 6: Practice with transformations. For each matrix $A$ below, (i) state the domain and codomain of $T_{A}(\vec{x})=A \vec{x}$, (ii) find $T_{A}\left(e_{1}\right), T_{A}\left(e_{2}\right)$, (iii) find $T_{A}(v), T_{A}(w)$, (iv) describe in a few words what the transformation is doing, $(i v)$ state whether $T$ is one-to-one and if not then find a vector $\vec{x}$ which maps to zero (verify with calculations that your choice maps to zero by plugging in $\vec{x},(\boldsymbol{v})$ state whether $T$ is onto and if not then find a vector $\vec{b}$ which is not in the range (and verify by showing that $A \vec{x}=\vec{b}$ is inconsistent), (vi) state $\operatorname{dimNul} A$ and $\operatorname{dimCol} A$, and finally (vii) give the matrix an appropriate "name" (fine to be silly name like, e.g., "the x-zero-er" for projection to y-axis"). For the problems below use

$$
e_{1}=\left[\begin{array}{l}
1 \\
0
\end{array}\right], \quad e_{2}=\left[\begin{array}{l}
0 \\
1
\end{array}\right], v=\left[\begin{array}{c}
-1 \\
2
\end{array}\right], \quad w=\left[\begin{array}{c}
2 \\
-3
\end{array}\right]
$$

(1a) $A=\left[\begin{array}{cc}-1 & 0 \\ 0 & 1\end{array}\right]$
(1b) $A=\left[\begin{array}{ll}1 / 2 & 1 / 2 \\ 1 / 2 & 1 / 2\end{array}\right]$
(1c) $A=\left[\begin{array}{ll}1 & 0 \\ 0 & 1 \\ 0 & 0\end{array}\right]$
(1d) $A=\left[\begin{array}{ll}2 & 0 \\ 0 & 1\end{array}\right]$
(1e) $A=\left[\begin{array}{ll}1 & 2 \\ 1 & 2\end{array}\right]$
(1f) $A=\left[\begin{array}{cc}\sqrt{2} / 2 & \sqrt{2} / 2 \\ -\sqrt{2} / 2 & \sqrt{2} / 2\end{array}\right]$
$(\mathbf{1 g}) A=\left[\begin{array}{ll}1 & 1 \\ 0 & 1 \\ 1 & 0\end{array}\right]$
(1h) $A=\left[\begin{array}{cc}1 & -1 \\ 1 & 1 \\ 0 & 0\end{array}\right]$
Next, for the problems below use

$$
e_{1}=\left[\begin{array}{l}
1 \\
0 \\
0
\end{array}\right], \quad e_{2}=\left[\begin{array}{l}
0 \\
1 \\
0
\end{array}\right], \quad e_{3}=\left[\begin{array}{l}
0 \\
0 \\
1
\end{array}\right], \quad v=\left[\begin{array}{c}
3 \\
-1 \\
2
\end{array}\right], \quad w=\left[\begin{array}{c}
0 \\
1 \\
-2
\end{array}\right]
$$

(2a) $A=\left[\begin{array}{lll}1 & 0 & 0 \\ 0 & 0 & 1\end{array}\right]$
(2b) $A=\left[\begin{array}{lll}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0\end{array}\right]$
(2c) $A=\left[\begin{array}{lll}0 & 2 & 1 \\ 0 & 3 & 0 \\ 3 & 1 & 0 \\ 1 & 1 & 2\end{array}\right]$
(2d) $A=\left[\begin{array}{lll}1 & 1 & 1 \\ 1 & 1 & 1\end{array}\right]$

Sal says: For the "name your matrix" this is a bit silly, and that's ok. Just come up with a creative name that makes sense to you, and don't worry about it too much!

Hint: For part (2c) above, an appropriate name could be 'the total mystery' or the 'crazy transformation', or maybe 'some kind of weird shear embedding'. It's not always easy to see what the matrix is doing! Don't overthink this one, please.

## Week 7: Transformations and the determinant.

Step 1: Sketch a parallelogram somewhere in $\mathbb{R}^{2}$ such that none of the vertices of the parallelogram lie on the origin. Label the vertices of the parallelogram $a, b, c, d$ and also label the coordinates $\left(x_{1}, x_{2}\right)$ for each of the four points. Label the parallelogram $S$ for shape.

Step 2b: Showing your work, compute the area area $(S)$ using the content of Section 3.3 , by finding a pair of vectors $\vec{v}_{1}, \vec{v}_{2}$ which determine the parallelogram from Step 1. Go back and label the vectors $\vec{v}_{1}, \vec{v}_{2}$ in your sketch from Step 1.
Step 2: Choose a (somewhat random) linear transformation $T: \mathbb{R}^{2} \rightarrow \mathbb{R}^{2}$ which can be anything but must satisfy:
(i) the transformation should not have a diagonal standard matrix,
(ii) the transformation should be invertible (one-to-one and onto),
(iii) the transformation should not be a single rotation or a single reflection*.
$\left({ }^{*}\right)$ it is ok for the transformation you pick to be a rotation followed by a reflection, for example. However, if your transformation is 'two rotations' or 'two reflections' but can be represented by a single rotation or single reflection, then you will lose points.
Step 3: Transform your shape $S$ from Step 1 using your transformation $T$ from Step 2. That is, sketch the image of $S$ in $\mathbb{R}^{2}$. Make a new sketch for this part and label the images $T(a), T(b), T(c), T(d)$ and give the new coordinates for all four points.
Step 3b: Showing all work, compute the area $\operatorname{Vol}(T(S))$ using the same method as Step $2 b$ by finding the images of $\vec{v}_{1}, \vec{v}_{2}$ under the transformation $T$. Label the images $T\left(\vec{v}_{1}\right), T\left(\vec{v}_{2}\right)$ in your sketch from Step 3.

Step 4: Compute $\operatorname{det} A$ where $A$ is the standard matrix of $T(\vec{x})=A \vec{x}$, and compare the areas $\operatorname{Vol}(S)$ and $\operatorname{Vol}(T(S))$ with the value of $\operatorname{det} A$. Write a general formula which relates these three quantities.

Week 8: MATLAB \#2 Basis of Eigenvectors and Markov chains. Note: see Sal's webpage.
Week 9: Three separate parts.

1. Find a $2 \times 2$ matrix $A$ with real entries with no real eigenvalues and show that it is correct by finding the characteristic polynomial and explaining why it has no real roots.
2. Find all eigenvalues and a corresponding eigenvector for each matrix below without calculations by thinking it out using the linear transformation's geometric interpretation. For each matrix, graph each eigenvector and it's image after the transformation as well as a random NON-eigenvector. Check your graph is accurate with matrix multiplication. Confirm your geometric intuition by finding the eigenvalues, algebraic and geometric multiplicity of each eigenvalue, and a basis of eigenvectors for each eigenspace (you may need to do this step first for any transformations that you can not visualize).
(a) $A=\left[\begin{array}{ll}0 & 0 \\ 0 & 0\end{array}\right], T_{A}$ is the zero transformation.
(b) $A=\left[\begin{array}{ll}1 / 2 & 1 / 2 \\ 1 / 2 & 1 / 2\end{array}\right], T_{A}$ is projection onto $x_{1}=x_{2}$ line.
(c) $A=\left[\begin{array}{cc}\cos (\theta) & -\sin (\theta) \\ \sin (\theta) & \cos (\theta)\end{array}\right], T_{A}$ is rotation by $\theta$. (and state the values of $\theta$ for which $A$ has (real) eigenvectors)
(d) $A=\left[\begin{array}{cc}0 & -1 \\ -1 & 0\end{array}\right], T_{A}$ is reflect about the line $x_{1}=-x_{2}$.
(e) $A=\left[\begin{array}{ll}1 & 0 \\ 0 & 3\end{array}\right], T_{A}$ is stretch in $x_{2}$-direction.
(f) $A=\left[\begin{array}{ll}1 & 1 \\ 0 & 1\end{array}\right], T_{A}$ is a shear.
(g) $A=\left[\begin{array}{lll}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0\end{array}\right], T_{A}$ is project to the floor.
(h) $A=\left[\begin{array}{ccc}0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1\end{array}\right], T_{A}$ is rotate by $90^{\circ}$ about the $z$-axis.
3. Pick any two transformations of $\mathbb{R}^{2}$ above other than (a), do one row operation to the matrix to obtain a matrix $B$ and compute the eigenvalues of the new matrix. Make sure to pick $A$ and do a row operation so that $B$ has different eigenvalues than $A$ (at least one eigenvalue should be different - this is not guaranteed). Compute the eigenvectors for each eigenvalue, are they the same or different? Finish the sentence below. When you do a single row operation to a matrix $A$ then the eigenvalues... and the eigenvectors ....

Week 10: note: This week's assignment doesn't have to be handwritten. (For 1pt) Write a paragraph which explains how eigenvectors/eigenvalues or some other topic from the course are used in a field which interests you. Be specific and put some thought into this. (For 2pt) If your paragraph is not essentially the thing on wikipedia about how bridges have something to do with eigenvalues, but actually give some details or original content then you get 2 pts instead of 1 pt. (For 3pt) If you also support your research with real math or alternately something creative. This has to include some mathematical content but can take any form whatsoever. Last year's submissions included a poem, several posters, a few slide-show presentations like using PowerPoint, etc., and quite a few of them actually were pretty decent research project results that I was quite impressed by, but I remember the poem the best; it was funny and it used the right math ideas about linear algebra to be funny, which essentially forces that the person understood the concepts. It was brilliant.

WARNING: Your submission must not be something already covered this semester, or about something we will cover later. So you can not write about the Google matrix, for instance. If you want to write about a topic that was cut but appeared previously then that's ok (e.g., Leontief, homogeneous coordinates, computer graphics.)

Please understand the point of this exercise, should you choose to do it: Pick any scientific discipline. I mean ANY. If it is scientific it's ok: how to build a bridge, how do design a new chemical, how to solve some hard algorithmic problem using computers (like how many stars the Netflix algorithm should predict for your enjoyment of the 1977 original version of Disney's Pete's Dragon, for example). Take 5 steps into your chosen scientific field and you will bump into linear algebra. That's the exercise. 1pt is essentially "write down in your own words what wiki has to say about it", 2pts is essentially "do something a little better but without any real math content", and then 3 pts is "a pretty good job explaining how a specific concept from linear algebra is used in a scientific field you are interested in", where it is up to the grader to determine if what you present with the math explaining qualifies for +3 pt .

Week 11: MATLAB \#3 Least-squares exploration - see Sal's website for instructions and supplementary documents.

Week 12: MATLAB \#4 Google PageRank - see Sal's website for instructions.
Week 13: Spectral decomposition and constrained optimization.
Part 1: Pick two random vectors in $\mathbb{R}^{3}$ (no zeros - few repeated numbers) and call them $\vec{u}$ and $\vec{v}$. Compute the matrix $\vec{u} \vec{v}^{T}$ which is size $3 \times 3$. Write out the entire sentence below, with the boxes, and fill them in with the appropriate word/phrase to complete the sentence.
The rows of the matrix $\vec{u} \vec{v}^{T}$ are scalar multiples of $\square$ with scalars that are $\square$.
Part 2: Let

$$
A=\left[\begin{array}{ccc}
1 & 4 & 6 \\
4 & -3 & 2 \\
6 & 2 & 2
\end{array}\right]
$$

Compute (a) an orthogonal decomposition $A=P D P^{T}$ where $D=\left[\begin{array}{ccc}9 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -3\end{array}\right]$, and (b) the spectral decomposition of $A$, namely express $A$ as the sum of rank 1 matrices

$$
A=\lambda_{1} \vec{v}_{1} \vec{v}_{1}^{T}+\lambda_{2} \vec{v}_{2} \vec{v}_{2}^{T}+\lambda_{3} \vec{v}_{3} \vec{v}_{3}^{T}
$$

where $P=\left[\vec{v}_{1} \vec{v}_{2} \vec{v}_{3}\right]$ so that $\vec{v}_{i}$ are the normalized eigenvectors of $A$. Last, (c) write down $Q_{A}(\vec{x})=\vec{x}^{T} A \vec{x}$ the quadratic form defined by $A$. You will analyze this quadratic form next.
Part 3: First (a) Write $Q_{A}(\vec{x})$ as a quadratic form with no cross terms $Q_{A}(\vec{x})=$ $Q_{D}(\vec{y})$ using the orthogonal diagonalization in Part 2. Then (b) find a unit vector $\vec{y}$ which attains the maximum $M=\max \left\{Q_{D}(\vec{y}):\|\vec{y}\|=1\right\}$, and (c) find a corresponding vector $\vec{x}$ which maximizes $Q_{A}(\vec{x})$. Finally, (d) find a vector $\vec{x}$ which attains the same value of as $Q_{D}(1,0,1)=12$ when plugged into $Q_{A}$, check your answer by plugging in the vector $\vec{x}$ into $Q_{A}(\vec{x})$ and make sure you get 12 as the output.

Week 14: MATLAB \#5 - see Sal's website for instructions and supplementary documents.

