Worksheet 5: Chapter 3 (Product, quotient, and chain rule)

1. Find any horizontal tangents in the interval $0 \leq x \leq 2\pi$.

(a) $y = x + \sin x$

(b) $y = x - \cot x$

2. Find the derivative of the function.

(a) $y = (2x + 4)^3(x + 1)^{-2}$

(b) $y = (1 - x)e^{x^3}$

(c) $r = 12(\sec\theta - \tan\theta)^{1/3}$

3. The height in feet of a ball above the ground t seconds after it is thrown is given by

$$s(t) = -4.9t^2 + 20t + 6.$$

(a) What is the height of the ball $t = 4$ seconds after it is released? What about $t = 2$ seconds after release?

(b) At the apex of the balls trajectory, it's velocity is momentarily zero. Find the time that this occurs.

(c) Find the second derivative of $s(t)$. What do you notice about the function $a(t) = s''(t)$? Interpret your answer in the context of the problem.

4. Find the derivative of $y = \dfrac{x^2 - 1}{x^3 + 1}$. What is the slope of the line tangent to the graph of the function at $x = 2$?

5. First simplify the expression, then take the derivative.

$$y = \frac{(2x-1)(x^2-3x)}{x^3}$$

6. Find $f''(x)$ where

$$f(x) = \frac{x^3+2}{x}.$$

7. Find the equation of the line that is tangent to the graph of $y = xe^x$ at $x = 1$.

Worksheet 6: Chapter 3 (Implicit differentiation and log derivatives)

1. Find the derivative $f'(x)$ two ways: *(first way)* first use properties of logs to simplify and then take the derivative, and *(second way)* take the derivative directly using the chain rule. Which way is easier for you?

$$f(x) = \ln\left(\frac{x^2}{1-x}\right)$$

2. Find the equation of the one of the lines tangent to the curve given by the equation

$$3xy - y^2 - x^3 = 1$$

at a point on the curve where $x = -2$. *There are two possible answers!* Illustrate the problem using software or sketch the situation.

3. Show that the point $(2, 4)$ lies on the curve $x^3 + y^3 - 9xy = 0$. Then find the tangent and normal to the curve at the point $(2, 4)$. Illustrate the example using software or make a sketch.

4. Use logarithmic differentiation to find the derivative of y with respect to t if

$$y = \left(\sqrt{t}\right)^t.$$

5. Use either a 30-60-90 or a 45-45-90 reference triangle to evaluate the given expression.

(a) $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

(b) $\csc^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

6. Use the graphs of the trig functions to evaluate the limits. If the limit does not exist write either DNE, $+\infty$ DNE, or $-\infty$ DNE whichever is the most appropriate.

(a) $\lim_{x\to 1} \sin^{-1} x$

(b) $\lim_{x\to \infty} \sec^{-1}(x)$

(c) $\lim_{x\to 1^-} \cos^{-1}(x)$

7. Use the inverse trig derivative identities to find y'.

(a) $y = \cos^{-1}(x^2)$

(b) $y = \sec^{-1}(5x)$

Worksheet 7: Chapter 3 (Related rates)

1. Water runs into a conical tank at the rate of 9 ft^3/min. The tank stands pointed downwards and has a height of 10 feet and the conical base radius is 5ft. How fast is the water rising when the water is 3 ft deep? *Recall: the volume of a cone is given by* $V = \frac{1}{3}\pi r^2 h$.

2. A balloon rises above a stationary point on the ground 500 ft from an observer. At a certain time the observers' viewing angle is $\pi/4$, and the angle is increasing at a rate of 0.14 rad/min. At this time, how high off the ground is the balloon? how fast is the balloon rising?

3. A 13-ft ladder is leaning against the wall of a house when its base starts to slip and slides away from the wall. By the time the base is 12 ft from the house, the base is moving at a rate of 5 ft/sec.

(a) At this moment, how fast is the side of the ladder sliding down the wall?

(b) At what rate is the area of the triangle formed by the ladder, the ground, and the wall changing at this moment?

(c) At what rate is the angle θ made by the ground and the ladder changing at that moment?

4. The radius r and height h of a right circular cylinder are related to the cylinder's volume V by the formula $V = \pi r^2 h$. This problem asks you to analyze how the volume of the cylinder changes if the radius r or the height h change over time.

(a) Suppose r is a constant so that $dr/dt = 0$. Relate dV/dt and dh/dt using the volume equation $V = \pi r^2 h$ and differentiation with respect to the time variable t.

(b) Suppose now that h is constant, so that dr/dt, but that the radius $r = r(t)$ of the cylinder changes over time. Find an equation relating dV/dt to dr/dt.

(c) Finally, find an equation which relates dV/dt to both dr/dt and dh/dt. Do not assume that either r or h is constant.

1. During an experiment the velocity of a particle at time t is given by (15 pts.)

$$v(t) = \frac{3t}{(1+t^2)^2}$$

for $t > 0$. How far has the particle travelled in the first 2 seconds of the experiment?

2. Solve the separable differential equation. (15 pts.)

$$\frac{dy}{dx} = x^2\sqrt{y}, \quad y > 0.$$

3. Find the value of the definite intergral. *Hint: try integration by parts.* (15 pts.)

$$\int_1^e x^3 \ln(x)\, dx$$

4. Find the area bounded by the two curves $y = x^3 - 2x^2 + 1$ and $y = 3x + 1$. (15 pts.)

5. Integrate using any method. (10 pts. each)

(a) $\displaystyle\int \tan(3x)\,dx$

(b) $\displaystyle\int (4x - 3 + \sec^2(x))\,dx$

6. Find the value. (10 pts. each)

7. $\displaystyle\int_0^{\pi/4} \sec(x)\, dx$

8. $\displaystyle\int_0^{1} \frac{1}{\sqrt{1-x^2}}\, dx$