Practice Exam 2

1. True or False questions.

(a) The matrix $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ corresponds to a system of linear equations with infinitely many solutions.

(b) Given two mutually exclusive events E and F, we have $P(E \text{ or } F) = P(E) + P(F)$.

(c) If E and F are independent events then $P(E \text{ and } F) = P(E) \cdot P(F|E)$.

(d) If I is the 3×3 identity matrix and A is any 3×3 matrix, then $AI = IA$.

(e) Roll a die and record the number and let E and F be the following events $E = \{2, 4, 6\}$ and $F = \{1, 3, 5\}$. Then the events E and F are independent.

2. Find the matrix product of AB and BA if

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \\ 4 & 1 \end{bmatrix} \quad \text{and } B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}.$$

3. Solve the system of linear equations with augmented matrix A given below. Use elementary row operations to obtain the rref (reduced row echelon form) of A and be precise in your answer. You should assume that the column variables are x, y, z in the usual order.

$$\left[\begin{array}{ccc|c} 2 & 0 & -2 & 6 \\ -1 & 2 & 3 & 7 \\ 1 & 2 & 1 & 13 \end{array}\right]$$

4. Consider an experiment where two fair dice are rolled and the sum of the two numbers are recorded. Let X be the sum of the two numbers which appear face up on the dice. Find the expected value and variance of X.

5. Suppose four fair die are rolled. What is the probability that at least one of the die shows either a 1 or a 2?

6. Consider the following two-stage experiment. First, we draw a card from a 52-card deck. If the card is a face-card then we flip a coin, and if it is not a face card then we roll a die. Find the probability that we end the sequence with a "6" on the die or with a "heads" on the coin.

7. Let X be a normally distributed continuous random variable with $\mu = 6$ and $\sigma = 2$. Find $P(X \leq 5)$ and $P(2.5 \leq X \leq 10)$.

8. A washing machine manufacturer knows that 2% of its machines break down in the first year. Estimate the probability of at least 15 out of 1000 washers breaking down in the first year.