Quiz 4

1. Let A, B be finite sets and $n \in \mathbb{N}$. Suppose there are n elements in A and n elements in B. How many one-to-one functions are there $f : A \to B$? Justify your answer. (8 pts.)

$A = \{a_1, \ldots, a_n\}$
$B = \{b_1, \ldots, b_n\}$

$f: \{a_1, \ldots, a_n\} \to \{b_1, \ldots, b_n\}$

$f(a_1) = b_i \to n$ choices for b_{i_1}
$f(a_2) = b_{i_2} \neq b_{i_1} \to n-1$ choices for b_{i_2} (can't be b_{i_1})
$f(a_3) = b_{i_3} \notin \{b_{i_1}, b_{i_2}\} \to n-2$ choices for b_{i_3} (can't be b_{i_1}, b_{i_2})
$\vdots$
$f(a_n) = b_{i_n} \to 1$ choice for b_{i_n} (forced by earlier choices)

mult rule:

$n * (n-1) * \ldots * 1 = \boxed{n!}$

2. A group of 18 students are surveyed to find out what subject they like the most.

12 said they prefer math class
8 said they prefer computer science
4 said they prefer both math and computer science

How many students preferred neither math nor computer science? (8 pts.)

$|U| = 18$
$|A| = 12$
$|B| = 8$
$|A \cap B| = 4$

$|A \cup B| = |A| + |B| - |A \cap B| = 12 + 8 - 4 = 16$

$|U| = |A \cup B| + |(A \cup B)^c|$ (add. rule)

$18 = 16 + x$

So $x = \#$ like neither $= \boxed{2}$

3. In my sock drawer I have three kinds of socks: athletic socks, dress socks, and casual socks. Each kind of sock has many pairs in my drawer, and all pairs of the same kind are identical, and there is no difference between left-socks and right-socks. How many individual socks must I pull out of the drawer to guarantee that I have a pair of the same kind? (4 pts.)

By pigeonhole, need smallest x such that

$$\left\lceil \frac{x}{3} \right\rceil = 2.$$

So $\boxed{x = 4}$.