Quiz 7

1. Consider the algorithm below with real inputs $a_1, \dots, a_n$ and x, and output s.

Input: $a_1, \dots, a_n, x \in \mathbb{R}$.
Procedure: Initialize $s = 0$.
Step 1: For $i = 1, \dots, n$,
For $j = 1, \dots, n$,
If $i \neq j$ and $a_i + a_j = x$, set s to 1.
Output: s.

(a) Find the output with input $a_i = 2^i$, $i = 1, \dots, 5$, and $x = 12$ and describe the relationship in general between the input and the output of the algorithm. That is, what does the algorithm *do*? (5 pts.)

Input: 2, 4, 8, 16, 32 $x = 12$
Output: $s = 1$ since $4 + 8 = 12$
(and 4, 8 are different a_i's)

The algorithm outputs $s = 1$ if two different a_i's add up to x, and $s = 0$ otherwise.

(b) Find an accurate bound on the total complexity of the algorithm. Is the complexity of the algorithm $O(n^2)$? Is the complexity $O(n^3)$? *Answer the three parts separately.* (7 pts.)

Comparisons $2n^2$
Arithmetic n^2
total $\boxed{3n^2}$

$3n^2 = O(n^2)$ yes ✓
$3n^2 = O(n^3)$ yes ✓

2. Use the definitions to prove that if $f = O(h)$ and $g = O(h)$ then $f + g = O(h)$. (8 pts.)

Proof Suppose $\exists c_1, c_2$ positive constants and $\exists n_1, n_2$ natural numbers such that

① $|f(n)| \le c_1 |h(n)|$ for $n \ge n_1$

and ② $|g(n)| \le c_2 |h(n)|$ for $n \ge n_2$

Then $|f(n) + g(n)| \le |f(n)| + |g(n)| \le c_1 |h(n)| + c_2 |h(n)|$ ① & ②, if $n \ge \max\{n_1, n_2\}$.
$= (c_1 + c_2)|h(n)|$
$\Delta \neq$ Set $n_0 = \max\{n_1, n_2\}$ and $C = c_1 + c_2$