Worksheet 2

1 Discuss the validity of the following argument.

> If I have to pay taxes then I'm unhappy or rich
> If I'm rich then I have a Ferrari
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> If I have to pay taxes then I'm unhappy and I have a Ferrari.

If it is not valid, make a change that turns it into a valid statement.

2 Consider the set $X = \{a, b, c, d\}$.
a) Write all subsets of X with less than two elements.
b) What is the number of subsets of X with less than three elements.
c) What is the number of subsets of X with at least three elements.
d) $|\mathcal{P}(\mathcal{P}(X))| = ?$

3 Prove that the following is right or give a counterexample.

$$(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$$

4 Let $A = \{a + b\sqrt{5} \,|\, a, b \in Q\}$ and $B = \{a + b\sqrt{10} \,|\, a, b \in Z\}$. Is it true that $A \subseteq B$? How about $B \subseteq A$?

5. Give an example of two sets that have five elements each and whose intersection has exactly three elements. How many elements does the union of your two sets have? How many answers to this question are possible?

6. Give an example of three sets whose number of elements are 3, 5, and 7 respectively. How many elements are possible to be in the intersection of all the sets? How many elements are possible to be in the intersection of **two** of the sets?