Quiz 2 (11 am)

1. Write the vector $b = \begin{bmatrix}1\\4\\1\end{bmatrix}$ as a linear combination of $v_1 = \begin{bmatrix}-1\\1\\2\end{bmatrix}$ and $v_2 = \begin{bmatrix}4\\1\\-5\end{bmatrix}$, or state that this is not possible. Clearly show your work and be clear about what is your answer. (10 pts.)

$$[v_1\ v_2\ |\ b] = \left[\begin{array}{cc|c} -1 & 4 & 1\\ 1 & 1 & 4\\ 2 & -5 & 1\end{array}\right]$$

$$\begin{matrix} \\ -R_1 \\ R_1+R_2 \\ 2R_1+R_3\end{matrix}\left[\begin{array}{cc|c} +1 & -4 & -1\\ 0 & 5 & 5\\ 0 & 3 & 3\end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -4 & -1\\ 0 & 1 & 1\\ 0 & 0 & 0\end{array}\right]$$

$$\sim \begin{matrix} \\ \\ 4R_2+R_1 \end{matrix}\left[\begin{array}{cc|c} 1 & 0 & 3\\ 0 & 1 & 1\\ 0 & 0 & 0\end{array}\right]$$

So $\boxed{3\begin{bmatrix}-1\\1\\2\end{bmatrix} + \begin{bmatrix}4\\1\\-5\end{bmatrix} = \begin{bmatrix}1\\4\\1\end{bmatrix}}$

self-check? ✓✓ good ✓

2. For each matrix below, determine if the matrix is in rref or not. If it is, state whether the associated system of linear equations has a unique solution, no solution, or infinitely many solutions. (1 pt. each part, 10 total)

(a) $\left[\begin{array}{cc|c} 0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{array}\right]$ rref/not rref unique/none/infinitely many

(b) $\left[\begin{array}{cc|c} 1 & 2 & -1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{array}\right]$ rref/not rref unique/none/infinitely many

(c) $\left[\begin{array}{cc|c} 0 & 1 & 0\\ 0 & 0 & 1\end{array}\right]$ rref/not rref unique/none/infinitely many

(d) $\left[\begin{array}{cc|c} 1 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 1\end{array}\right]$ rref/not rref unique/none/infinitely many

(e) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{array}\right]$ rref/not rref unique/none/infinitely many