Quiz 2 (12 pm)

1. Write the vector $b = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$ as a linear combination of $v_1 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, or state that this is not possible. Clearly show your work and be clear about what is your answer. (10 pts.)

$$[v_1\ v_2 \mid b] = \left[\begin{array}{cc|c} -1 & 1 & 5 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{array}\right] \sim \begin{matrix} \\ 4R_1+R_2 \\ R_1+R_3 \end{matrix} \left[\begin{array}{cc|c} -1 & 1 & 5 \\ 0 & 7 & 21 \\ 0 & 3 & 9 \end{array}\right]$$

$$\sim \begin{matrix} -R_1 \\ \\ \\ \end{matrix} \left[\begin{array}{cc|c} 1 & -1 & -5 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

So $\quad -2\begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$ $\quad A\vec{x}=b$

self-check? ✓✓ good ✓.

2. For each matrix below, determine if the matrix is in rref or not. If it is, state whether the associated system of linear equations has a unique solution, no solution, or infinitely many solutions. (1 pt. each part, 10 total)

(a) $\left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right]$ rref/not rref unique/none/infinitely many

(b) $\left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$ rref/not rref unique/none/infinitely many

(c) $\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$ rref/not rref unique/none/infinitely many

(d) $\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{array}\right]$ rref/not rref unique/none/infinitely many

(e) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ rref/not rref unique/none/infinitely many