Quiz 4 (12 pm)

1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which associates to each $\mathbf{x} \in \mathbb{R}^2$ the vector obtained from $\mathbf{x}$ by first reflecting $\mathbf{x}$ about the horizontal x-axis and then rotating $\mathbf{x}$ by $90°$ clockwise. Find the standard matrix A of T as well as the image $T\left(\begin{bmatrix}1\\1\end{bmatrix}\right)$. *Hint: the first column of A is $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$ and the second column of A is $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$.* (4 pts. ea.)

Idea:

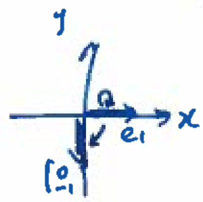
$A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$

ANS.

$$A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \qquad T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}-1\\-1\end{bmatrix}.$$

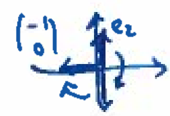
Work.

$e_1 = \begin{bmatrix}1\\0\end{bmatrix}$



$T(e_1) = \begin{bmatrix}0\\-1\end{bmatrix}$

$e_2 = \begin{bmatrix}0\\1\end{bmatrix}$



$T(e_2) = \begin{bmatrix}-1\\0\end{bmatrix}$

2. Determine whether the given vectors are linearly independent or linearly dependent. If the vectors are linearly dependent find a non-trivial linear combination of the vectors which give the zero vector. (8 pts.)

$$\underset{v_1}{\begin{bmatrix}3\\-1\\0\end{bmatrix}},\ \underset{v_2}{\begin{bmatrix}1\\1\\2\end{bmatrix}},\ \underset{v_3}{\begin{bmatrix}7\\-1\\2\end{bmatrix}}$$

Solve $Ax = 0$

$$\begin{bmatrix} 3 & 1 & 7 \\ -1 & 1 & -1 \\ 0 & 2 & 2 \end{bmatrix} \overset{3R_2+R_1}{\sim} \begin{bmatrix} 0 & 4 & 4 \\ -1 & 1 & -1 \\ 0 & 2 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{array}{c} \begin{array}{ccc} x & y & z=r \end{array} \\ \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$x = -2r$
$y = -r$
$z = r$ free

$r = -1$

$$\boxed{\text{Linearly dependent} \quad 2v_1 + v_2 = v_3}$$

3. True or False section. (1 pt. each)

T/F If A is a 4×3 matrix with 3 pivots, then the columns of A are linearly independent.

T/F If $Ax = 0$ has the trivial solution, then the columns of A are linearly independent.

False T/F If the columns of A are linearly independent, then $Ax = b$ has a unique solution.
could be inconsistent

T/F The linear transformation with standard matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ rotates vectors in $\mathbb{R}^2$ by $90°$ ~~counter-clockwise~~.
Clockwise