Warmup 6 Probabilities: Counting

1. An experiment consists of rolling a fair die twice. Find the probability that the dice have a sum of 5 or the first die shows a number less than 3.

Solution: Let $E = \{\text{sum of } 5\}$ and let $F = \{\text{the first die shows less than } 3\}$. Then $E = \{(1,4),(2,3),(3,2),(4,1)\}$ and $F = \{(1,1),(1,2),...,(1,6),(2,1),(2,2),...,(2,6)\}$. Since the dice are fair, each outcome has probability $\frac{1}{36}$. Thus, $Pr(E) = \frac{4}{36}$ and $Pr(F) = \frac{12}{36}$. Note also that $Pr(E \cap F) = \frac{2}{36}$, so by inclusion/exclusion, we have:

$$Pr(E \cup F) = \frac{4}{36} + \frac{12}{36} - \frac{2}{36} = \frac{7}{18}.$$

2. There are four organizations that want to hold an information session in the student center sometime next week (Sunday-Saturday, 7-day week). What is the probability that at least two of the organizations are scheduled to hold their information session on the same day?

Solution: This problem works like the birthday problem.

$$Pr(\geq 1 \quad match) = 1 - Pr(no \quad matches) = 1 - \frac{P(7,4)}{7^4}.$$

3. You and your two best friends order three different sub sandwiches at the new Subway. However, the cashier forgets who ordered which sub and hands them out at random. What is the probability that exactly one of you will receive the correct sandwich?

Solution: There are only three ways to have exactly one person receive the same sandwich: choose one of the three friends, then the other two sandwiches are swapped. Thus:

$$Pr(exactly \quad 1 \quad correct) = \frac{3}{3!} = \frac{1}{2}.$$