Warmup 7 Conditional Probabilities

1. The probability of making an "A" in finite math is 0.3. The probability that a student attends class regularly is 0.6, and the probability that a student makes an "A" and attends class regularly is 0.4. Are the events $E = \{\text{makes an "A"}\}$ and $F = \{\text{attends class regularly}\}$ independent? Explain your answer mathematically.

Solution: Here, $Pr(E) = 0.3$, $Pr(F) = 0.6$, and $Pr(E \cap F) = 0.4$. Note that

$$Pr(E) \cdot Pr(F) = (0.3) \cdot (0.6) \neq Pr(E \cap F).$$

Thus, the events are dependent.
Alternatively, you can find $Pr(E|F) = \frac{0.4}{0.6} = \frac{2}{3}$, which is not $Pr(E)$, so the probability of an "A" increases for students with regular attendance.

2. A pair of dice are rolled once and the numbers on the faces are recorded. Find the probability that the dice showed doubles if it is known that the sum is an even number.

Solution: Even sums are 2, 4, 6, 8, 10, 12. There are $1+3+5+5+3+1 = 18$ outcomes with even sums. There are six possible doubles, all of which have an even sum. Let $E = \{\text{even sum}\}$ and let $F = \{\text{doubles}\}$. Then:

$$Pr(E) = \frac{1}{2}, \quad Pr(F) = Pr(E \cap F) = \frac{1}{6}.$$

Then:

$$Pr(F|E) = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}.$$