Worksheet 06/22/2016

Problem 1 *Find the limit with L'Hopital's Rule*

1. $\lim\limits_{x \to 0^+} x \ln x$
2. $\lim\limits_{x \to 0} \dfrac{x 2^x}{2^x - 1}$
3. $\lim\limits_{x \to 1^+} \left(\dfrac{1}{x-1} - \dfrac{1}{\ln x} \right)$
4. $\lim\limits_{x \to 0^+} (\ln x - \ln \sin x)$

5. $\lim\limits_{x \to \infty} (\ln x)^{1/x}$
6. $\lim\limits_{x \to 0^+} \left(1 + \dfrac{1}{x} \right)^x$
7. $\lim\limits_{x \to 0^+} x (\ln x)^2$
8. $\lim\limits_{x \to 0^+} \sin x \ln x$

Problem 2 *Evaluate the following integrals.*

1. $\displaystyle\int \dfrac{2x^3 - 2x^2 + 1}{x^2 - x} dx$

2. $\displaystyle\int \dfrac{\cos x \, dx}{\sin^2 x + \sin x - 6}$

3. $\displaystyle\int \dfrac{x^4}{x^2 - 1} dx$

4. $\displaystyle\int \dfrac{(x-2)^2 \arctan(2x) - 12x^3 - 3x}{(4x^2 + 1)(x - 2)^2} dx$

5. $\displaystyle\int \dfrac{dx}{x^{3/2} - \sqrt{x}}$

6. $\displaystyle\int \dfrac{dx}{x\sqrt{x + 9}}$

Problem 3 *Evaluate the following integrals*

1. $\displaystyle\int_0^2 \dfrac{dx}{\sqrt{1 - x^2}}$
2. $\displaystyle\int_0^\infty \dfrac{dx}{(1 + x)\sqrt{x}}$
3. $\displaystyle\int_0^1 x \ln x \, dx$
4. $\displaystyle\int_{-1}^4 \dfrac{dx}{\sqrt{|x|}}$
5. $\displaystyle\int_0^\infty \dfrac{dx}{(x + 1)(x^2 + 1)}$

Problem 4 *Check if the following integrals converge.*

1. $\displaystyle\int_0^1 \dfrac{dt}{t - \sin t}$
2. $\displaystyle\int_0^\pi \dfrac{dt}{\sqrt{t} + \sin t}$
3. $\displaystyle\int_0^2 \dfrac{dx}{1 - x}$
4. $\displaystyle\int_2^\infty \dfrac{dx}{\ln x}$

5. $\displaystyle\int_{e^e}^\infty \ln(\ln x) dx$
6. $\displaystyle\int_0^\infty \dfrac{dx}{\sqrt{x^6 + 1}}$
7. $\displaystyle\int_1^\infty \dfrac{dx}{e^x - 2^x}$
8. $\displaystyle\int_1^\infty \dfrac{\sqrt{x + 1}}{x^2} dx$