Worksheet 6/28/2016

Problem 1 *Solve the following 1st order linear ODEs with or without initial values*

1. *Derive the formula you can always use*

$$\frac{dy}{dx} + P(x)y = Q(x)$$

2. *Use the above one to solve the following IVPs*

$$(1)\ y' + (\tan x)y = \cos^2 x,\ x \in (-\frac{\pi}{2}, \frac{\pi}{2}),\ y(0) = 0$$
$$(2)\ \theta\frac{dy}{d\theta} - 2y = \theta^3 \sec\theta \tan\theta,\ \theta > 0,\ y(\pi/3) = 2$$
$$(3)\ (t+1)\frac{dx}{dt} - 2(t^2+t)x = \frac{e^{t^2}}{t+1},\ t > -1,\ x(0) = 5$$

Problem 2 *Check if the following sequences converge or not when n goes to infinity. If yes, find the limit.*

$$(1)\ 2, 2 + \frac{1}{2}, 2 + \frac{1}{2 + \frac{1}{2}}, 2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}, \cdots$$

$$(2)\ \sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}, \cdots$$ (*Hint: Show that $a_n < 2$ and is monotone*)

(3) $a_n = \dfrac{n!}{n^n}$ (4) $a_n = \dfrac{n!}{2^n \cdot 3^n}$ (5) $a_n = \sqrt{n}\sin\dfrac{1}{\sqrt{n}}$ (6) $a_n = n(1 - \cos\dfrac{1}{n})$ (7) $a_n = \sqrt[n]{n^2+n}$

(8) $a_n = 1 + (-1)^n$ (9) $a_n = \left(\dfrac{n}{n+1}\right)^n$ (10) $a_n = n - \sqrt{n^2 - n}$