1. (14 pts) Prove that $C^\infty_0(\mathbb{R}^n)$ is dense in $W^{1,p}(\mathbb{R}^n)$ for $1 \leq p < \infty$.

2. (13 pts) Let $U = \{x \in \mathbb{R}^n : x_1, \ldots, x_n > 0\}$. Find an extension operator for $W^{1,p}(U)$.

3. (14 pts) Let $U$ be a bounded, open subset of $\mathbb{R}^n$ with a $C^1$ boundary. Show that there exists a constant $C$ such that

$$
\int_U |Du|^2 dx \leq C \left( \int_U u^2 dx \right)^{\frac{1}{2}} \left( \int_U |D^2u|^2 dx \right)^{\frac{1}{2}}
$$

for all $u \in H^2(U) \cap H^1_0(U)$.

4. (14 pts) Let $U$ be an open subset of $\mathbb{R}^n$ and let $1 \leq p \leq \infty$. Let $u, v \in W^{1,p}(U) \cap L^\infty(U)$. Show that $uv \in W^{1,p}(U) \cap L^\infty(U)$ and that for $1 \leq i \leq n$

$$(uv)_x = u_x v + uv_{x_i}.$$