Tentative Final Review Outline

Final is take-home from Tuesday April 28th to Wednesday April 29th. You can spend any time on the problems within these 48 hours. Books, notes, calculators and online resources are allowed, but you can not communicate with anybody on the exam.

Final covers all materials. You expect to see both theoretical and programming problems. The level of difficulty is similar to HW problems.

Errors
- Floating point representation, chopping and rounding, unit round, the maximum accuracy possible, underflow and overflow
- Significant digits, relative error, propagation of errors

Finding roots
- The bisection method, the Newton’s method, the Secant method
- Definition of the speed of convergence, the speed of convergence for the bisection method and the Newton’s method. The speed of convergence for the Secant method is not required.

Interpolation
- Polynomial interpolation: Lagrange form, Newton interpolation formula
- Error of polynomial interpolation
- Chebyshev polynomials and Chebyshev nodes
- Divided differences and the table for Newton interpolation
- Hermite interpolation and the table for Newton interpolation

Splines
- What is a linear spline? Given a set of knots and function values, construct a linear spline.
- What is a cubic spline? Given a set of knots and function values, construct a cubic spline.
- Determine some parameters to make a given function a cubic spline, such as HW 3 Problem T1.

Least squares and orthogonal polynomials
- What are least squares?
- Compute inner product and the least squares solution, such as HW 3 Problem T3.
- Orthogonal polynomials, such as the Chebyshev polynomial and Legendre polynomial. What are they and why are they orthogonal?
Interpolation in higher dimensions

- What is two-dimensional Lagrange interpolation and Shepard interpolation?
- Theorem 2 in Section 6.10 of Kincaid and Cheney. Proof is not required.

Numerical integration

- Trapezoidal rule and its error analysis.
- Simpson’s rule and its error analysis.
- Newton-Cotes integration formula. Error analysis is not required.
- What is Gaussian quadrature? How to find a Gaussian quadrature?

Solving systems of linear equations

- Gauss elimination with pivoting: compute $PA = LU$ by hand or with code
- Richardson Method, Jacobi Method, Gauss Seidel Method: Program these methods. As a theoretical question, when do these methods converge?

Computing eigenvalues and eigenvectors

- Power method to find the eigenvalue with the largest magnitude and the associated eigenvector: program it and what can you say about the convergence?
- Inverse power iteration to find the eigenvalue closest to $\mu$ and the associated eigenvector: program it and what can you say about the convergence?
- Rayleigh Quotient iteration to find some eigenvalue and the associated eigenvector: program it and what can you say about the convergence?

All Homework problems are required.