HW 2 – 4640, Spring 2020
Instructor: Wenjing Liao

• HW 2 is due on Monday Feb 10 at the beginning of the class.
• Please write your solutions independently, and explain your numerical results. Your code is to be turned in on Canvas.
• You are strongly encouraged to type out your solutions using latex.

1. Theoretical problems

T1: Write the Lagrange and Newton forms of the interpolating polynomial for the following data:

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

Write both polynomials in the form $ax^2 + bx + c$ to verify that they are identical.

T2: If we interpolate the function $f(x) = e^{x-1}$ with a polynomial $p$ of degree 12 using 13 nodes in $[-1, 1]$, what is a good upper bound for $|p(x) - f(x)|$ on $[-1, 1]$?

T3: Let $f(x) = x^7 + x^4 + 3x + 1$. Compute $f[2^0, 2^1, \ldots, 2^7]$ and $f[2^0, 2^1, \ldots, 2^7, 2^8]$.

T4: Prove that if $f$ is a polynomial of degree $k$, then for $n > k$,

$$f[x_0, x_1, \ldots, x_n] = 0.$$

T5: Use Newton divided difference method to compute a polynomial $p(x)$ of degree at most 4 such that $p(0) = p'(0) = 0$, $p(1) = p'(1) = 1$ and $p(2) = 1$.

T6: Find a piecewise linear interpolation for the data:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>1.4</td>
<td>0.6</td>
<td>1.0</td>
<td>0.6</td>
<td>1.0</td>
</tr>
</tbody>
</table>
T7: Find values of \((a, b, c, d)\) that make the following function a cubic spline
\[
p(x) = \begin{cases} 
x^3 & x \in [-1, 0] 
a + bx + cx^2 + dx^3 & x \in [0, 1]
\end{cases}
\]

2. Programming problems

P1: Interpolate the following functions on \([-5, 5]\) with \(n+1\) nodes by polynomial \(p_n\) of order no more than \(n\).

\[
f(x) = \frac{1}{1 + x^2} \\
g(x) = e^{-x^2}
\]

(1) Try \(n+1\) equally spaced nodes. How does the error \(\|f - p_n\|_\infty\) or \(\|g - p_n\|_\infty\) scale as \(n\) increases? Explain your results.

(2) Does the interpolation of these two functions have a similar appearance? Why?

(3) Try \(n+1\) Chebyshev nodes. How does the error \(\|f - p_n\|_\infty\) or \(\|g - p_n\|_\infty\) scale as \(n\) increases? Explain your results.

P2: Write your own code to perform the algorithm of divided differences and Newton interpolation. Test your code on the following data. Give the polynomial of degree 4 that interpolates

\[
\begin{array}{cccc}
x & \text{-1} & 0 & 1 & 1 & 3 \\
\hline
f(x) & 2 & 1 & 0 & y & z
\end{array}
\]

where \(y\) and \(z\) are the last two digits of your student ID.