Runge’s example

\[ f(x) = \frac{1}{1 + 25x^2} \quad x \in [-1, 1] \]
Runge’s example

Equidistant: \( x_j = -1 + \frac{2}{n} j \) for \( j = 0, 1, \ldots, n \)
Runge's example

Equidistant: \[ x_j = -1 + \frac{2}{n} j \quad \text{for} \quad j = 0, 1, \ldots, n \]
Runge’s example

Chebyshev: $x_j = \cos \left( \frac{2j + 1}{2(n + 1)} \pi \right)$ for $j = 0, 1, \ldots, n.$
Runge’s example

Chebyshev: \( x_j = \cos \left( \frac{2j + 1}{2(n + 1)} \pi \right) \) for \( j = 0, 1, \ldots, n \).
Runge’s example

Equidistant

Chebyshev

Interpolation error: $\|e_n\|_{\infty} = \|f - p_n\|_{\infty}$
Runge’s example

\[ f(x) = \frac{1}{1 + 25x^2} \quad x \in [-1, 1] \]
Runge's example

Piecewise linear

\[ f(x) \]
\[ v(x) \text{ with } M = 5 \]
\[ v(x) \text{ with } M = 11 \]
Runge’s example

Piecewise quadratic
Runge’s example

Piecewise interpolation error

![Graph showing the error in piecewise interpolation for different numbers of subintervals. The graph plots the infinity norm of the difference between the function and its piecewise interpolation against the number of subintervals. The error decreases as the number of subintervals increases, with different orders of magnitude for piecewise linear, quadratic, and an unspecified higher-order interpolation. The graph includes annotations for $O(h^2)$ and $O(h^3)$, but the order for the unspecified interpolation is denoted as $O(h^?)$.](image_url)
Runge’s example

Piecewise interpolation error

$$\| f(x) - v(x) \|_\infty$$

- Piecewise linear
- Piecewise quadratic
- Piecewise "can you guess"?

- $O(h^2)$
- $O(h^3)$
- $O(h^6)$

Round-off error!

Number of subintervals

$M$