Math 1502 Quiz1 Summer 2007

Solutions for section A1, A2

Problem 1
Find the least positive integer n so that the $n-th$ degree Taylor Polynomial $P_n(x)$ in x approximates $sin0.3$ to within 0.01.
Solution: The $2n+1-th$ reminder of $f(x) = sinx$ is

$$R_{2n+1}(x) = \frac{f^{2n+3}(c)}{(2n+3)!}x^{2n+3}$$

We want $|R_{2n+1}(0.3)| < 0.01$.
Since $|(sinx)^{(n)}| \leq 1$ for all n and x, we only need to find the least n such that $\frac{0.3^{2n+3}}{(2n+3)!} < 0.01$.
It's easy to see $n = 0$ is the least integer, and $2n+1 = 1$. So the $1st$ degree Taylor Polynomial of $sinx$ approximates $sin0.3$ to within 0.01.

Problem 2
Find $lim_{x\to+\infty}(1+e^{2x})^{1/x}$.
Solution: This is the ∞^0 form, so we take the natural log first.

$$\begin{aligned} lim_{x\to+\infty}ln(1+e^{2x})^{1/x} &= lim_{x\to+\infty}\tfrac{1}{x}ln(1+e^{2x}) \\ &= lim_{x\to+\infty}\tfrac{ln(1+e^{2x})}{x} \\ &= lim_{x\to+\infty}\tfrac{2e^{2x}}{1+e^{2x}} \qquad (applying\ \ L'Hospital's\ \ Rule) \\ &= lim_{x\to+\infty}\tfrac{4e^{x}}{2e^{x}} \qquad (applying\ \ L'Hospital's\ \ Rule\ \ again) \\ &= 2 \end{aligned}$$

So $lim_{x\to+\infty}(1+e^{2x})^{1/x} = e^2$.