Math 1502 Quiz3 Summer 2007

Solutions for section A1, A2

Problem 1
Find the general solution of $y' - 2y = 1$.
Solution: This is a 1st order linear equation with standard form $y' + P(x)y = Q(x)$.
Here $P(x) = -2$, $Q(x) = 1$, so the integrating factor $H(x) = \int P(x)dx = \int -2dx = -2x$.
The general solution is

$$\begin{aligned} y(x) &= e^{-H(x)}(\int e^{H(x)}Q(x)dx + C) \\ &= e^{2x}(\int e^{-2x}dx + C) \\ &= e^{2x}(-\tfrac{1}{2}e^{-2x} + C) \end{aligned}$$

Problem 2
Solve the initial value problem: $y' = \frac{e^{x-y}}{1+e^x}$, $y(0) = 0$.
Solution: This is a separable equation.

$$\frac{dy}{dx} = y' = \frac{e^x}{1+e^x}e^{-y}$$

so

$$e^y dy = \frac{e^x}{1+e^x}dx$$

take the integral on both sides:

$$\begin{aligned} e^y &= \int \tfrac{e^x}{1+e^x}dx + C \\ &= ln(1+e^x) + C \end{aligned}$$

Plug in the initial value $y(0) = 0$ to calculate the constant C:

$$e^0 = ln(1+e^0) + C \Longrightarrow C = 1 - ln2.$$

Thus $e^y = ln(1+e^x) + 1 - ln2$.