MATH3012
Review for Test 1

Test 1 will cover the following sections: 1.1–1.4, 4.1–4.5, 5.1–5.3 and 5.5. The test problems will be similar to the problems assigned and examples given in the class. A formula sheet is allowed (but no example problems). No calculator please. You might find the following review material helpful.

- Chapter 1 deals with counting problems. There are two basic principles we use when we count: the addition principle and the multiplication principle.

  Given a complicated problem, you should be able to break the problem into disjoint cases or independent stages, then solve each case or stage, and finally add or multiply the answers from the cases or stages.

- To find answers for cases or stages in a complicated problem, we often need to solve selection problems or arrangement problems (or equivalently, problems in which order is relevant or irrelevant). There are two ways to look at the selection and arrangement problems, the selection model and the distribution model.

- In the selection model, we count the number of selections of r objects from n distinct objects (or n types of objects) with or without repetitions.

  If the *order* is not important or you are just selecting objects, then you are dealing with a selection problem. If the objects cannot be repeated (or each type has exactly one object), then the number of selections is $\binom{n}{r}$. If the objects can be chosen with unlimited repetitions, then the number of selections is $\binom{n+r-1}{r}$. If some objects have limited repetitions (for example, object A can be repeated at most 10 times, or equivalently, type A has 10 objects), then you need to break the original counting problem into cases or stages so that in each of these cases or stages, either all objects cannot be repeated or all have unlimited repetitions.

  If the *order* is important or you are selecting objects to arrange, then you are dealing with an arrangement problem. If the objects cannot be repeated (or each type has exactly one object), then the number of arrangements is $P(n, r)$. If the objects can be chosen with unlimited repetitions, then the number of arrangements is n^r. If we are arranging i_1 objects of type 1, i_2 objects of type 2, …, and i_n objects of type n, then the total number of arrangements is $\frac{r!}{i_1!i_2!\cdots i_n!}$ (this is called multinomial coefficient). All other arrangement problems can be reduced to cases or stages of the above three types.

- In the distribution model, we count the number of distributions of r objects into n distinct boxes.

  If the objects are *all identical*, then the order is not important. That is, we are dealing with a selection problem. If each box can receive at most one object, then the number of distributions is $\binom{n}{r}$. If each box can receive any number of objects, then the number of distributions is $\binom{n+r-1}{r}$. For all other selection problems, you need to reduce them to cases or stages of the above two types.

If the objects are *all distinct*, then you are dealing with an arrangement problem. If each box can receive at most one object, then the number of distributions is $P(n, r)$. If each box can host any number of objects, then the number of distributions is n^r. If we distribute exactly i_j objects to jth box, for $j = 1, \cdots, n$, then the number of distributions is $\frac{r!}{i_1!i_2!\cdots i_n!}$. All other arrangement problems can be reduced to cases or stages of the above three types.

- You should be able to find the number of integer solutions of equations and inequalities.

- Section 4.1 deals with mathematical induction. There are two (equivalent) versions of induction, weak induction and strong induction.

  You should be able to use induction to prove statements about natural number n.

- Sections 4.3 and 4.4 study division algorithm and Euclidean algorithm.

  You should be able to use division algorithm to convert a decimal number to a number to a given base. Given positive integers m and n, you should be able to find $gcd(m, n)$, and express it as a linear combination of m and n.

- Sections 5.1 and 5.2 deal with relations. Functions are a special type of relations, and bijections are a special type of functions. If there is a bijection between finite sets A and B, then $|A| = |B|$.

  You should be able to determine whether a given relation is a function or a bijection, and to verify your conclusions.

- Section 5.5 is about the Pigeonhole Principle. It is an easy statement, but relevant problems could be quite hard to solve.

  You should be able to use the Pigeonhole Principle to solve some simple problems at the level no more difficult than the examples and homework problems.