MATH3012, Fall, 2008
Review for Test 2

Test 2 will cover the following sections: 8.1–8.5, 9.1, 9.2, 9.4, 9.5, 10.1–10.4. The test problems will be similar to the problems assigned and examples given in the class. A formula sheet is allowed; however, examples and definitions should not be included.

Chapter 8. You are expected to use the inclusion–exclusion formulas to solve counting problems, in particular, those with restrictions which are difficult to deal with using "elementary" methods.

You are expected to compute rook polynomials. You are also expected to use rook polynomials to solve arrangement problems with forbidden positions (by describing the forbidden positions as a chessboard).

Chapter 9. You are expected to find generating functions of sequences, extract coefficients in generating functions, and solve counting problems using generating functions.

There are two types of generating functions: *ordinary* and *exponential*. It is very **important** to decide which type of generating functions you need to use when solving a problem. Ordinary generating functions can be used to solve problems in which order does not matter (use common sense and look for key words such as "selection" and "distribution of identical objects"); and exponential generating functions can be used to solve problems in which order does matter (use common sense and look for key words such as "arrangements", "sequences", and "distribution of distinct objects").

When using generating functions to solve problems, you find the corresponding generating function and simplify it using formulas, and then extract the coefficient of x^k for an appropriate integer k. Note that in the exponential generating function case, you need to multiply the coefficient of x^k by $k!$.

You are also expected to use ordinary generating functions to find formulas for $a_0+\cdots+a_n$, where a_n is a polynomial of n (for example, $a_n = n^2$). This is dealt with in Section 9.5. If $f(x)$ is the generating function of $\{a_n\}$, then $f(x)/(1-x)$ is the generating function of $a_0+\cdots+a_n$. Hence, all you need to do is to find $f(x)$ and then find the coefficient of x^n in $f(x)/(1-x)$. To find $f(x)$, you need to use and manipulate the formula for $1/(1-x)$ (such as taking derivatives).

Chapter 10. You are expected to find recurrence relations for certain problems, solve certain linear recurrence relations, and use recurrence relations to solve certain problems. Do not forget to find initial conditions for the recurrence relation you found.

When finding recurrence relations, you need to break a problem into cases or steps in a way that you can reduce the size of the objects you are counting (for example, the length of a binary sequence). The way to break a problem to cases or steps depends on the particular problem, but the "rule of thumb" is to look at the possibilities of the first (or last) "choice" in the objects you are counting.

To solve a linear recurrence relation, first you find the general solution a_n^g of the homogeneous part, then you find a particular solution a_n^p for the original recurrence relation, and finally set $a_n = a_n^g + a_n^p$ and use initial conditions to solve for unknown coefficients.

To find a_n^g, you need to find the characteristic roots of the recurrence relation, and then find a_n^g according to the rules. To find a_n^p, you look at the inhomogeneous part $f(n)$. If $f(n)$ is a polynomial of degree k, and 1 is a characteristic root with multiplicity m, then a_n^p is a polynomial of degree $k+m$; and you need to set $a_n = a_n^p$ (and so, $a_{n-1} = a_{n-1}^p$, etc.) and use the original recurrence relation to find a_n^p precisely. If $f(n) = r^n$, then $a_n^p = Ar^n$ if r is not a characteristic root, and $a_n^p = An^m r^n$ if r is a characteristic root of multiplicity m, and finally find A using the original recurrence relation. If $f(n)$ is a combination of r^n and a polynomial, then you can guess a_n^p as a combination of r^n and a polynomial.

Finally, there is a relation between generating functions and recurrence relations. You are expected to be able to use ordinary generating functions to solve certain recurrence relations.