Workshop on Graph Theory and Combinatorics in Memory of Robin Thomas

in combination with Atlanta Lecture Series in Graph Theory and Combinatorics XXVI

August 19 – 21st

Georgia Institute of Technology

Atlanta, United States

Organizing Committee

Matt Baker (Georgia Institute of Technology)
Prasad Tetali (Carnegie Mellon University)
Xingxing Yu (Georgia Institute of Technology)

Schedule

Friday, August 19th

Friday, August 19th		
Time	**Speaker**	**Title**
9:00-9:30		Opening Remarks
9:40-10:30	Bojan Mohar	Game of cops and robber on geodesic spaces
Coffee Break		
10:50-11:40	Martin Loebl	Aspects of Kasteleyn Orientations
Lunch Break		
13:00-13:50	Guoli Ding	Induced subgraphs and well-quasi-ordering
14:00-14:50	Sang-il Oum	Building the hierarchy of graph classes
Coffee Break		
15:10-16:00	James Oxley	Generalizations of cographs for graphs and matroids
16:10-16:35	Andrea Jimenez	Grid subdivisions, wall minors and treewidth in planar graphs
16:45-17:10	Bogdan Oporowski	Unavoidable large 2-connected induced subgraphs
17:30-18:30	In Skiles Classroom Building	Cold Water Challenge
Dinner		

Saturday, August 20th

Friday, August 19th		
Time	**Speaker**	**Title**
9:00-9:50	Maria Chudnovsky	Induced subgraphs and tree width
Coffee Break		
10:10-11:00	Paul Wollan	Explicit bounds in graph minors
11:10-12:00	Zixia Song	Every graph with no $\mathcal{K}_8^{-4}$ minor is 7-colorable
Lunch Break		
13:00-13:50	Luke Postle	Generalizing and Localizing the Four Color Theorem
14:00-14:50	Zdeněk Dvořák	3-coloring triangle-free graphs on surfaces
15:00-15:30	Youngho Yoo	A unified Erdős-Pósa theorem for cycles in graphs labelled by multiple abelian groups
Coffee Break		
16:00-16:50	Chun-Hung Liu	Homomorphism counts in robustly sparse graphs
17:00-17:50	Bojan Mohar	On crossing-critical graphs
Dinner		

Sunday, August 21st

Sunday, August 21st		
Time	**Speaker**	**Title**
9:00-9:50	Theo Molla	Minimum color degree thresholds for rainbow subgraphs
10:00-10:50	Jinyoung Park	Thresholds
Coffee Break		
11:10-11:35	Rose McCarty	A combinatorial game for monadic stability
11:45-12:35	Dan Král	Spectral method in extremal combinatorics
End of Conference		

Title & Abstracts

Induced subgraphs and tree width

Maria Chudnovsky
Princeton University

Tree decompositions are a powerful tool in structural graph theory; they are traditionally used in the context of forbidden graph minors. Connecting tree decompositions and forbidden induced subgraphs is a subject that has recently gained some attention. Several constructions show that it is probably very difficult to characterize induced subgraph obstructions to bounded tree width, but over the last years several sufficient conditions have been found. We will discuss recent progress in this direction, using a variety of techniques. We will also touch on the question of characterizing the obstructions for the tree width to be logarithmic in the number of vertices of the graph.

Induced subgraphs and well-quasi-ordering

Guoli Ding
Louisiana State University

A set of graphs is called an *antichain* if no member is an induced subgraph of another member. A family of graphs is called *hereditary* if every induced subgraph of a member remains a member.

Let $\mathcal{G}_n$ be the hereditary family of graphs that do not contain $K_{n,n}$ as a subgraph. Let $\mathcal{S}_m$ be the hereditary family of graphs G for which there exists a set U of at most m vertices such that every component of $G \backslash U$ is a path. We prove the existence of a function $m = m(n)$ with the following property: for any hereditary family $\mathcal{G} \subseteq \mathcal{G}_n$, $\mathcal{G}$ contains an infinite antichain if and only if $\mathcal{G} \cap \mathcal{S}_m$ contains an infinite antichain.

3-coloring triangle-free graphs on surfaces

Zdeněk Dvořák
Charles University

A famous theorem of Grötzsch states that every planar triangle-free graph is 3-colorable. In a joint project with Dan Král̆ and Robin Thomas, we extended this result to all other surfaces, by providing an approximate description of minimal obstructions for 3-colorability of embedded triangle-free graphs and using it to answer various structural and algorithmic questions. In my talk, I will survey some recent developments on this topic, including an exact characterization of 3-colorability of toroidal triangle-free graphs and applications of nowhere-zero flows to design practical algorithms for the problem.

Grid subdivisions, wall minors and treewidth in planar graphs

Andrea Jiménez
Universidad de Valparaíso

Minors, subdivisions and treewidth are classical notions that appear for example in the seminal characterization of planar graphs by Kuratowski and in the celebrated Graph Minor Theorem by Robertson and Seymour. The importance of grid/wall minors comes from one of the results of Robertson and Seymour, which roughly claims that a graph of large treewidth necessarily contains a large grid/wall minor. The concept of treewidth is fundamental in algorithmic graph theory since many problems which are hard to solve in general, can be efficiently solved when restricted to classes of graphs with bounded treewidth.

In this talk, we discuss the computational complexity of the Minor Problem, the Subdivision Problem and the Treewidth Problem with input: grids/walls and planar graphs. Surprisingly, some of these problems are still open. We describe two reductions which prove that the respective GridSubdivision Problem and the WallMinor Problem are NP-complete problems.

This is joint work with Tina Janne Schmidt and with Carla Lintzmayer and Maycon Sambinelli.

Spectral method in extremal combinatorics

Daniel Král'
Masaryk University, Brno, Czech Republic

We will present two recent applications of spectral arguments in extremal combinatorics. The first concerns a quantitative version of Ramsey's Theorem. A graph G is common if the random 2-edge-coloring of a complete graph asymptotically minimizes the number of monochromatic copies of G among all 2-edge-colorings. The notion of common graphs can be traced back to the work of Erdős from the 1960s. In particular, Erdős conjectured that every complete graph is common, which was disproved by Thomason in the 1980s.

A classification of common graphs remains a challenging open problem. Sidorenko's Conjecture, one of the most significant open problems in extremal graph theory, implies that every 2-chromatic graph is common. While examples of 3-chromatic common graphs were known for a long time, the existence of a 4-chromatic common graph was open until 2012. Using spectral arguments in the setting of graph limits, we establish the existence of connected common graphs with an arbitrarily large chromatic number.

The second application concerns a classical problem on maximizing the density of a fixed subgraph. Specifically, we study the asymptotic behavior of the maximum number of cycles of a given length in a tournament: let $c(k)$ be the limit of the ratio of the maximum number of cycles of length k in an n-vertex tournament and the expected number of cycles of length k in the random n-vertex tournament, when

n tends to infinity. We will show that $c(k) = 1$ if and only if k is not divisible by four, which settles a conjecture of Bartley and Day. Moreover, if k is even but not divisible by four, then every extremal tournament is quasirandom.

The talk will include results obtained with different groups of collaborators, including Andrzej Grzesik, László Miklós Lovász, Jonathan A. Noel, Sergey Norin, Jan Volec and Fan Wei.

Homomorphism counts in robustly sparse graphs

Chun-Hung Liu

Texas A&M University

For a fixed graph H and for arbitrarily large host graphs G, the number of homomorphisms from H to G and the number of subgraphs isomorphic to H contained in G have been extensively studied when the host graphs are allowed to be dense. This talk addresses the case when the host graphs are robustly sparse. We determine, up to a constant multiplicative error, the maximum number of subgraphs isomorphic to H contained in an n-vertex graph in any fixed hereditary graph class with bounded expansion. This result solves a number of open questions and can be generalized to counting the number of homomorphisms from H.

Aspects of Kasteleyn Orientations

Martin Loebl

Charles University

Kasteleyn (Pfaffian) orientations belonged to basic research interests of Robin. I will review some of their recognised features and secrets from discrete math, complexity and statistical physics.

A combinatorial game for monadic stability

Rose McCarty

University of Warsaw

We characterize graph classes which are monadically stable, in the model-theoretic sense, via a simple combinatorial game. Monadically stable classes generalize classes with a “locally” forbidden minor; in fact the definitions coincide on classes which are “sparse”. A major motivation for this line of research is a theorem that Robin proved with Zdeněk Dvořák and Dan Král’ about classes of bounded expansion.

This is joint work with Pilipczuk, Siebertz, Ossona de Mendez, Toruńczyk, Przybyszewski, Sokołowski, Mählmann, Gajarský, and Ohlmann.

On crossing-critical graphs

Bojan Mohar

Simon Fraser University

A graph is k-critical for the crossing number if its crossing number is at least k, but removing any edge decreases the crossing number below k. Hlineny proved in in 2003 that k-crossing-critical graphs have bounded path-width. So, what else can one say about the structure of crossing-critical graphs for a fixed k? It turns out that there is much more, and this will be the main focus of the presentation.

The main results presented in the talk are joint work with Zdenek Dvorak and Petr Hlineny.

Game of cops and robber on geodesic spaces

Bojan Mohar

Simon Fraser University

The game of Cops and Robber is traditionally played on a finite graph. The speaker will introduce and analyze the game that is played on an arbitrary geodesic space (a compact, path-connected space endowed with intrinsic metric). In particular, this defines the game on any metric hypergraph. The main question of interest is how the structural conditions on the hypergraph influence the strategies to capture the robber.

Minimum color degree thresholds for rainbow subgraphs

Theo Molla

University of South Florida

Let $G = (V, E)$ be a graph on n vertices and let $c : E \to \mathbb{N}$ be a coloring of the edges of G. The color degree $d^c(v)$ of a vertex $v \in V$ is the number of distinct colors that appear on the edges incident to v. We let $\delta^c(G) = \min_{v \in V}\{d^c(v)\}$ be the minimum color degree of G. In 2013, H. Li proved that if $\delta^c(G) \geq (n+1)/2$, then G contains a rainbow triangle and this is tight as witnessed by a properly edge-colored balanced bipartite graph. In this talk, we will explore generalizations and extensions of this result. In particular, for $\ell \geq 4$, we will discuss the minimum color degree threshold for the existence of a rainbow ℓ-clique. This is joint work with Andrzej Czygrinow and Brendan Nagle.

Unavoidable large 2-connected induced subgraphs

Bogdan Oporowski
Louisiana State University

A classic result of Ramsey states that for every positive integer n, every sufficiently large graph contains an induced K_n or $\overline{K_n}$. Among the many extensions of Ramsey's Theorem there is an analogue for connected graphs: for every positive integer n, every sufficiently large connected graph contains an induced K_n, $K_{1,n}$, or P_n. In this talk, I will present an analogue for 2-connected graphs, which states that for every integer n exceeding two, every sufficiently large 2-connected graph contains one of the following as an induced subgraph: K_n, a subdivision of $K_{2,n}$, a subdivision of $K_{2,n}$ with an edge between the two vertices of degree n, and a well-defined structure similar to a ladder. This is Joint work with Sarah Allred and Guoli Ding.

Building the hierarchy of graph classes

Sang-il Oum
IBS Discrete Mathematics Group & KAIST

We will survey the classification of graph classes in terms of the transductions in monadic second-order logic. Blumensath and Courcelle (2010) characterized that every class of graphs is equivalent by transductions of the monadic second-order logic of the second kind to one of the following: class of all trees of height n for an integer n, class of all trees, class of all paths, and class of all grids. They conjectured that there is a similar linear hierarchy of graph classes in terms of the monadic second-order logic of the first kind. We will discuss how a recent theorem of the speaker with O-joung Kwon, Rose McCarty, and Paul Wollan (2019) on the vertex-minor obstruction for shrub-depth and a theorem of the speaker with Bruno Courcelle (2007) on graphs of large rank-width and logical expression of vertex-minors solve some subproblems of their conjecture.

Generalizations of cographs for graphs and matroids

James Oxley
Louisiana State University

The class of cographs or complement-reducible graphs is the class of graphs that can be generated from K_1 using the operations of disjoint union and complementation. We define 2-cographs to be the graphs we get by also allowing the operation of 1-sum. By analogy, we introduce the class of binary comatroids as the class of matroids that can be generated from the empty matroid using the operations of direct sum and taking complements inside of binary projective space. This talk will explore the properties of 2-cographs and binary comatroids. The main results characterize these classes in terms of forbidden induced minors. This is joint work with Jagdeep Singh.

Thresholds

Jinyoung Park
Stanford University

Thresholds for increasing properties of random structures are a central concern in probabilistic combinatorics and related areas. In 2006, Jeff Kahn and Gil Kalai conjectured that for any nontrivial increasing property on a finite set, its threshold is never far from its "expectation-threshold," which is a natural (and often easy to calculate) lower bound on the threshold. In this talk, I will present recent progress on this topic. Based on joint work with Huy Tuan Pham.

Generalizing and Localizing the Four Color Theorem

Luke Postle
University of Waterloo

The famous Four Color Theorem states that every planar graph is 4-colorable. Here we survey work by Robin, myself and others on how to generalize and localize the Four Color Theorem including recent developments on local algorithms and generalizations of coloring such as list coloring and correspondence coloring.

Every graph with no $\mathcal{K}_8^{-4}$ minor is 7-colorable

Zi-Xia Song
University of Central Florida

Hadwiger's Conjecture from 1943 states that every graph with no K_t minor is $(t-1)$-colorable; it remains wide open for all $t \geq 7$. For positive integers t and s, let $\mathcal{K}_t^{-s}$ denote the family of graphs obtained from K_t by removing s edges. We say that a graph G has no $\mathcal{K}_t^{-s}$ minor if it has no H minor for every $H \in \mathcal{K}_t^{-s}$. Jakobsen in 1971 proved that every graph with no $\mathcal{K}_7^{-2}$ minor is 6-colorable. In this talk, we consider the next step and present our recent work that every graph with no $\mathcal{K}_8^{-4}$ minor is 7-colorable. Our result implies that H-Hadwiger's Conjecture, suggested by Paul Seymour in 2017, is true for every graph H on eight vertices such that the complement of H has maximum degree at least four, a perfect matching, a triangle and a cycle of length four. This is joint work with Michael Lafferty.

Explicit bounds in graph minors

Paul Wollan

University of Rome

Robertson and Seymour proved an approximate characterization of all graphs excluding some fixed graph H as a minor, a result which has had an enormous impact on the field with numerous applications in graph theory and theoretical computer science. The proof is notable for its complexity; moreover, the proof does not give explicit bounds on the parameters involved.

We present recent work yielding new and simplified proofs for the main results in the graph minor structure theory. Beyond simplifying the results, we also for the first time give explicit bounds on the parameters involved. Joint with Ken-ichi Kawarabayashi and Robin Thomas.

A unified Erdős-Pósa theorem for cycles in graphs labelled by multiple abelian groups

Youngho Yoo

Texas A&M University

Erdős and Pósa showed in 1965 that cycles obey an approximate packing-covering duality. While odd cycles do not satisfy such a duality, Reed proved that the only obstruction is the presence of a large Escher wall. In this talk we discuss generalizations of these results. Namely, in undirected group-labelled graphs, we prove a characterization of the subsets of allowable values for which the family of allowable cycles satisfy the Erdős-Pósa property and characterize the obstructions, under some additional assumptions on the structure of the set of allowable values. This recovers many known results in the area and resolves a question of Dejter and Neumann-Lara from 1987 on characterizing when cycles of length l mod m satisfy the Erdős-Pósa property. Joint work with Pascal Gollin, Kevin Hendrey, O-joung Kwon, and Sang-il Oum.
